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Productivity analysis and functional specification of Pakistani textile industry

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textile industry**

Abbasi, Jani Alam, Ph.D.

Iowa State University, 1992

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Ann Arbor, MI 48106**

Productivity analysis and functional specification

of Pakistani textile industry

by

Jani Alam Abbasi

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**Iowa State University
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1992

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CHAPTER I. INTRODUCTION

As stated by Markly Roberts in his speech (1957),

Productivity improvement is not an end in itself. It is a means to an end. In fact it is a means to achieve a variety of social as well as economic goals. Economic goals and economic efficiency are by no means the only goals of any society, and it would be wrong to allow such goals, important though they may be, to be exclusive or overriding goals of a society.

Some major goals of a developing society, as well as a developed society contingent upon growth of productivity, are:

1. maximum freedom and dignity,
2. full employment,
3. sufficient and equal distribution of income,
4. adequate shelter and food,
5. good health and decent environment,
6. equal opportunity and enjoyable life,
7. democratic structure of society, industry, and community.

All of these above mentioned factors contribute to a better quality of life and standard of living which are achieved by the process of industrialization, with the net result of growth in productivity.

Throughout the world, it is believed that a better standard of living can only be achieved through the process of industrialization. The United Nations study of industrial growth states that industry is in itself a highly dynamic activity. The income per person engaged in industry is normally substantially higher than in agriculture. Also, it tends to exercise a dynamic impact on other sectors of a country's economy.

Among many factors, such as political, social, environmental and economical, which influence the process of industrialization, economical consideration is a top priority of any country, to be addressed properly and with positive approach.

From the cluster of economical considerations for industrialization and industrial growth, one aspect (and maybe most important) is to evaluate the performance of industry. Industrial performance is based on an input/output relationship of complex nature. A direct result of industrial performance is reflected in the standard of living of a particular country. A higher performance industry has full capacity utilization and is more productive with better employment and better living opportunity, as compared to one with low performance. To evaluate the performance improvements based on productive capacity of the industry, it is imperative to analyze the growth of industrial productivity. It is usually believed that the level of productivity growth of developing countries is low when compared to developed countries, but it is not yet completely known what the actual differences are, how they are caused, and the way to monitor them most effectively. It is believed that the causation of productivity level is the direct result of factors such as:

- (a) Optimal choice of technology,
- (b) Capital/Labor employed for cost/effectiveness,
- (c) The type and level of technical and managerial (organizational) know-how, and
- (d) Timing of implementation.

Pakistan is a developing country with a lesser developed economy and lower standard of living compared to the other industrialized and emerging industrialized countries. The

process of economic development or growth requires a set of institutions, attitudes and incentives that will assure continuity of growth. High annual rates of savings, continued improvements in methods of production, a steady provision of managerial, personnel and technical skills and a streamlined administrative organization make economic growth a quasi-automatic process.

To raise the standard of living, three factors which account for developmental trends and activities must be explored. These factors are:

1. technological,
2. economic, and
3. social.

The net effect of these factors is to reduce the cost of creating real wealth of the country. As the industrial sector is supposed to be a major contributor to the country's wealth, the analysis of its performance over a whole time period and/or at a specific point in time is of paramount importance for planners and policy makers. The living standard and the quality of life is directly proportional to the general economic well-being in one's country. Reduction in the cost of manufacturing and raising the productivity of manufacturing industries are the top priorities of every country in the world.

Changes in productivity are basically related to the following factors:

- (a) Capital and/or equipment,
- (b) Labor,
- (c) Implementive capability/Technical know-how/Technical stock of knowledge,
- (d) Organizational and managerial awareness,

- (e) Size and scale economics,
- (f) Optimal lot size and inventory policy,
- (g) Interindustry shifts and quality of resources, and
- (h) Availability of domestic resources.

Capital and/or Equipment

This factor determines how much capital is invested and in what form or capacity. Is it an effective investment or not; is there any need for addition, and if there is how it will be financed, i.e., by borrowing, by retained earnings or by new stock issue. In the case of equipment, is it running over/under capacity, is the equipment what is needed for prudent operation, do the operating conditions and maintenance schedules accord with the manufacturer specifications? Productivity will be affected positively with efficient employment of factors; otherwise it will decline.

Labor

Labor productivity is based upon the following:

1. Basic education,
2. Social environment,
3. Skill and Training,
4. Working conditions,
5. Incentives and motivation,
6. Management attitude, and
7. Overall image of the entity (country/industry).

Implementive Capability/Technical Know-how/Technical Stock of Knowledge

An advanced and high technology is worthless and will depress productivity, if an adequate amount of technical know-how or implementive capability is not available in the country (industry) on the same lines of action as desired by the High Technology.

Experience of both developed and developing countries shows that without technical knowledge, the performance of advanced technology is a failure.

Organizational and Managerial Awareness

Organizational and managerial awareness is supposed to be the backbone of any country (industry) for its growth and development. Proficient leaders (managers/organizers) always lead the country (industry) to the road of success and achievements. Proper coordination and cooperation of different sectors (departments) of a country's economy is the most important job of the leader (manager). This eventually leads to greater productivity and economic growth of the country. On the other hand, mismanagement leads to the collapse of the country/industry. Some sensitive manager/leaders, and lucky they are, grasp the ailing signs of the country/industry situation in time and take a sharp turn by implementing new rules and policies, which have already been tested by others, to save their country/industry from disaster.

Size and Scale Economies

The economies and diseconomies of scale depend on the size of the plant in operation. As will be discussed later in detail, that factor depends upon the elasticity of scale and size.

Optimal Lot Size and Inventory Policy

The optimal lot size and inventory policy will decrease the cost of production and avoid any shortages that can occur at the peak demand of the product.

Interindustry Shifts and Quality of Resources

Changes in the quality and quantity of resource allocation and/or substitution by the management can affect the cost of production. The poor quality substitute or non-optimal allocation of the resources can ruin the image of the industry and create financial problems leading to bankruptcy. Uneven allocations within departments will cause some schedule, labor and maintenance problems. This also results in less productivity.

Availability of Domestic Resources

The domestic resource for inputs will be far more cheaper and cost effective, compared to the imported one. The exploitation of domestic resource usage will help the industry and the country in productivity performance and economic well-being of the society.

Another area to be researched by the management is the area of material requirement planning or manufacturing resource planning (MRP) for increasing industrial productivity. Traditionally it is accepted that out of 100% of the time spent in the shop by a job, only 1.5% of the time the actual operation is done on the job, while 98.5% of the time the job is either moving or waiting for the operation. This is a very important consideration for a keen and sensitive management.

The economic growth of a country is proportional to the rate at which productivity increases or decreases. The western world is moving into a postindustrial society on the basis of their productivity growth. It is trying to compete fairly and successfully for a fair share in the global market place for consumer goods. A company can only operate successfully in a national/international market place if it can respond instantaneously to the changing market requirement without the loss of economical, functional and aesthetic capabilities of the product. Hence productivity should not diminish even when the demand variation is differential.

In general, the quality of life and economic well being is related indirectly to the cost of creating wealth, i.e., the cost accrued by the industrial sector or manufacturing industries. The lesser the cost of manufacturing, the higher the level of productivity will be of an industrial concern, and the country will be more prosperous. To date, very little has been done in this area. There are four studies worth mentioning, out of which three are dealing with the general manufacturing sector of Pakistan, and the fourth one sheds some light on the textile manufacturing and expresses an urgent need for another thorough study (1989).

This study is concerned with an analysis of the productivity growth of Pakistan's large scale textile industry. The large scale textile manufacturing industry is of paramount importance to the country because it is the predominant subsector for export as well as domestic consumption. The second reason is because of its linkage to the agriculture industry, the output of which is the input of this industry. Both these industries are labor

using. In a country like Pakistan where capital is scarce, it is very important to explore the capital saving possibilities in the industrial sector because of abundant labor.

As the value added by large scale manufacturing industries (textiles inclusive) has declined from .30 to .16 over the period of 1970-71 to 1980-81, and employment from .48 to .41 in the same period. It is the general objective of this study to undertake a systematic analysis of textiles manufacturing trends for improving the productivity level, which will help improve the economic well being of the people and the country.

Productivity can be measured partially with respect to individual factors as the ratio of total output to the individual classes of input and most importantly of labor. It is also measured with respect to all classes of input as the ratio of total output to total factors input, which is also known as multifactor productivity. These terms are used interchangeably in the literature. It is the residual of forces (factors) which are usually difficult to include in capital and labor productive stocks. This can be a mixture of both neutral and non-neutral technological changes, improvements in organizational and managerial structure, control of inventory and production flow process, changes in the quality of labor skills, material flow, material requirement and material handling techniques, changes in top executives policies and overall planning of the organization. Practically productivity is much more than a technological change, and this study will focus attention on the aspect of this input in total factor productivity modeling.

Multifactor productivity is measured as the ratio of real product to the associated real factor costs, labor and non labor. The weights are changed periodically to reflect

changes in the structure of production and in relative prices of outputs and of labor and capital inputs.

To analyze industrial growth, productivity and technology in the Pakistani large scale textile industry, the concept of aggregate production functions and duality of cost functions and procedure for their estimation will be utilized. The production function is a mathematical notation which shows the relationship between inputs and outputs of any particular system.

The system functional form introduced by Cobb-Douglas (CD) [1928] has an important impact on estimation of functional relationship for production or cost function. Basically, the whole effort was diverted to estimate the substitution effect between inputs. Usually, the substitution effect is demonstrated by the elasticity of substitution and denoted by σ . The principle of substitutability suggests that it is possible to produce a constant output level with a variety of input combination, only for those functions in which inputs are substitutable. There are many different definitions of substitution elasticity. Hicks (1963) offered the definition for two input x_1 and x_2 in the mathematical form as:

$$\sigma_{12} = \frac{d(x_2/x_1)/d(f_1/f_2)}{(x_2/x_1)/(f_1/f_2)} \quad \text{where } f_1 = \partial f / \partial x_1, f_2 = \partial f / \partial x_2 \quad \text{and } d(x_2/x_1)/d(f_1/f_2) \quad \text{Equation 1.1}$$

is given by the ratio of rotational change of the input ratios to the marginal rates of technical substitution, i.e., $\Delta k / \Delta t$ (see Figure 1.1). It can be readily seen from the curve that it is a measure of length of the arc CD or curvature of the isoquant.

In case of cost function σ_{12} between x_1 and x_2 gives the response of the ratio of the two factor demands when their price ratio changes, while output and other input

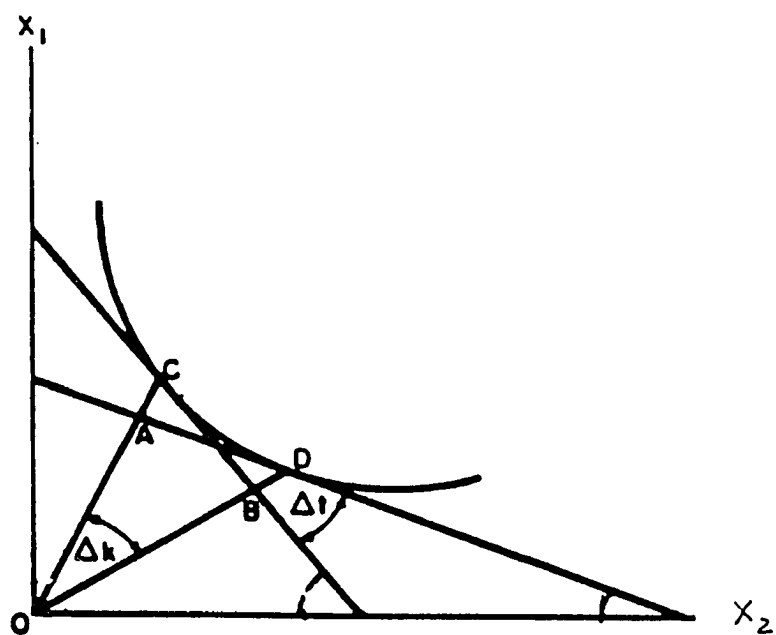


Figure 1.1 Elasticity of substitution (Chambers, 1990, p. 31)

quantities are held constant. In the logarithmic form it is

$$\sigma_{12} = \partial \ln(x_1/x_2) / \partial \ln(w_1/w_2), \quad w_i = \text{input price} \quad \text{Equation 1.2}$$

and measures the price induced change in the rate x_1/x_2 , conditional to other factors. It is also known as direct elasticity of substitution.

Another equivalent mathematical expression for σ is

$$\sigma_{12} = \frac{-f_1 f_2 (x_1 f_1 + x_2 f_2)}{x_1 x_2 (f_1^2 f_2 - 2 f_1 f_2 f_{12} + f_2^2 f_1^2)}, \quad \text{where } f_i = \frac{\partial f}{\partial x_i} \text{ and } f_{ij} = \frac{\partial^2 f}{\partial x_i \partial x_j} \quad \text{Equation 1.3}$$

The matrix form of the Equation 1.2 is

$$\frac{x_1 f_1 + x_2 f_2}{x_1 x_2} \cdot \frac{F_{12}}{F}, \quad \text{where } F = \begin{vmatrix} 0 & f_1 & f_2 \\ f_1 & f_{11} & f_{12} \\ f_2 & f_{12} & f_{22} \end{vmatrix} \quad \text{Equation 1.4}$$

is the bordered Hessian determinant of the production function, and F_{12} is the cofactor of f_{12} , i.e.,

$$F_{12} = \begin{vmatrix} 0 & f_1 \\ f_2 & f_{12} \end{vmatrix} = -f_1 f_2. \quad \text{Equation 1.5}$$

If production function is twice continuously differentiable, then Young's theorem implies that $f_{12} = f_{21}$ which is a symmetry condition.

$$\text{or } \frac{d \ln(x_2/x_1)}{d \ln(f_1/f_2)} = \frac{d \ln(x_1/x_2)}{d \ln(f_2/f_1)}. \quad \text{Equation 1.6}$$

For production function and for cost function $\ln(f_1/f_2)$ is replaced by $\ln(w_1/w_2)$. Hick's elasticity is also called direct elasticity of substitution and denoted by σ_{ij}^D (if $1,2$ is replaced by i,j) and it can be interpreted as short run or instantaneous, because it measures the

degree of substitutability between input i and j while all other inputs are held constant

[all $x_k (k \neq i, j) = \text{const}$].

The second definition of elasticity by Allen (1938) and Uzawa (1962) is the generalized form of Equation 1.4

$$\sigma_{ij} = \frac{\Sigma_i x_i f_i}{x_i x_j} \cdot \frac{F_{ij}}{F} \text{ (replacing 1,2 with } ij) \quad \text{Equation 1.7}$$

where

$$F = \begin{vmatrix} 0 & f_1 & f_2 & \cdots & f_n \\ f_1 & f_{11} & f_{12} & \cdots & f_{1n} \\ f_2 & f_{21} & f_{22} & \cdots & f_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ f_n & f_{n1} & \cdots & \cdots & f_{nn} \end{vmatrix} \quad \text{Equation 1.8}$$

and F_{ij} is the cofactor of f_{ij} . Both σ_{ij}^D and σ_{ij} are symmetric measure of degree of substitutability and are equal in the case of two input production/cost functions.

Economic goods are said to be complementary if $\sigma_{ij}^D = \sigma_{ij}^D < 0$ and substitute if $\sigma_{ij}^D = \sigma_{ij} > 0$.

The third type is Morishima elasticity, which can be written as:

$$\begin{aligned} \sigma_{ij}^M &= \left(\frac{f_i}{x_i} \cdot \frac{F_{ij}}{F} \right) - \left(\frac{f_j}{x_j} \cdot \frac{F_{ij}}{F} \right) = \frac{f_j x_j}{f_i x_i} \left[\frac{x_i f_i}{x_i x_j} \frac{F_{ij}}{F} - \frac{x_j f_j}{x_j x_j} \frac{F_{ij}}{F} \right] \\ \sigma_{ij}^M &= \frac{f_j x_j}{f_i x_i} (\sigma_{ij} - \sigma_{jj}), \end{aligned} \quad \text{Equation 1.9}$$

This shows that σ_{ij}^M is not symmetric -ie- $\sigma_{ij} \neq \sigma_{ji}$. Also this shows that a pair of goods can be Allen complements ($\sigma_{ij}^D < 0$), where they are Morishima substitute σ_{ij}^M , in the case

of two inputs while the Allen substitutes are always Morishima substitutes. In the case of multi inputs, the asymmetry of the Morishima measure of elasticity ($\sigma_{ij}^M \neq \sigma_{ji}^M$) has important implications in classifying the inputs as complements or substitutes and points out the somewhat arbitrary nature of any elasticity of substitution. The Arrow-Chenery-Minhas-Solow (1961) and Cobb-Douglas (CD) generalization for constant elasticity of substitution (CES) and Halten, Carter and Hocking (1957), Bruno (1968), Ravankar (1971), and Sato and Hoffman (1968) for variable elasticity of substitution (VES), allowed arbitrarily constant value of the elasticity different from unity, but in the multifactors case the partial elasticities of substitution have to be equal to the same constant.

Mukerji (1963), Gorman (1965) and Hanoch (1971) have proposed a functional form yielding partial elasticities that can differ along the isoquant pairwise, but still they are in constant ratio.

The above mentioned functional forms cannot be approximated to either second order numerical (Taylor Series) or second order differential approximation, and hence are not flexible.

Based on the duality concept between cost and production functions, the production function can be traced back from the cost function if information about the cost function is available. Mcfadden (1978a) has termed cost functions as "sufficient statistics" for the technology (production function). Cost function is the minimum cost of producing a given output level during a given time period expressed as function of input prices and output. Many flexible functional forms have been developed under the auspices of

duality in the context of multifactors inputs such as Diewert (1971) generalized Leontief (GL), Christensen, Jorgensen and Lau (1971) Translog (TLOG) and particularly Diewert (1974a) Quadratic square rooted (QSR), which has the dual of same form. These flexible forms neither restrict σ 's nor ratios of σ 's.

In specifying the functional form for production analysis, it is important that the relationship should be estimable with few prior restrictions on the technology. Estimability usually determines the choice of form, and if the form is parameterized according to economic theory (homogeneity, convexity etc.), duality guarantees the existence of a unique dual function. The primary goal of applied production analysis is empirical measurement of the economically relevant information that exhaustively characterizes the behavior of economic agents. In the case of smooth technologies (those that are twice-continuously differentiable) this (behavior of economic agents) includes the value of the objective function (the level of cost or output), the gradient of the function (the derived demands or MRTS) and Hessian (the matrix of derived demand elasticities). The goal is to select a form that should be rich enough in parameters to take care of all these above mentioned effects without imposing any prior constraints on these effects. For example, if one is investigating the production function (primal technology), one should have all the relevant information in term of production level, marginal productivities and matrix of elasticities of substitution. The choice of the form is important because any conclusion drawn is only valid within the confines of that model, and key parameter estimates like elasticities of substitution may be over or under the desired fair value. Therefore, it is recommended that form should be as general as

possible with as little as possible restriction on the ultimate outcome. The range of analysis and classical statistical tests are conducted under the presumption that the general form (model) is valid.

Validation of the model is very important since it is very difficult to discriminate among many flexible forms on theoretical grounds because all forms can be interpreted as second order approximations to an arbitrary twice differentiable function of the variables involved. The only way to discriminate is based on ease of estimation and appropriateness of stochastic representation of the model (form). Thus, choosing a functional form requires judgement as well as knowledge, and is more a craft than a science.

In the case of cost functions for estimation of share equations, the translog form will be the best choice because the share equation system is linear in parameters. On the other hand, if an investigator utilizes a cost function linear in input prices, the best choice will be Leontief. Goodness of fit tests can be used for discriminating among the flexible forms as used by Berndt, Darrough and Diewert in the Bayesian framework and White Sell (1985) in the TLOG case. Kiefer's (1975) development of flexible form, based on the Box-Cox (1974) transformation, has been modified and extended by Khaled (1978). The present study is heavily based on the generalized Box-Cox (GBC) functional form used by Khaled (1978). The GBC form allows parametric tests of homotheticity, homogeneity and symmetry. Parks' (1971) and Woodland's (1975) extensions of homothetic GL functions are more convenient than GBC, but it is not possible to impose homotheticity and homogeneity separately. Contrary to Diamond and McFadden (1975),

the GBC formulation is capable of identifying the bias in technical change and the elasticities of substitution simultaneously by utilizing time series data alone. This results in parametric estimation of total factor productivity (TFP), free from the errors of cost minimizing behavior, as compared to the traditional residual measure in terms of rate of growth in real output minus the rate of growth in real input, which is only justifiable in the case of constant returns to scale technology. While in the variable case the TFP includes the effect of technical change as well as economies of scales in the residual measure of productivity.

The GBC representation of functional form is rich in parameters (more free parameters) and thus suitable for analyzing interrelationship among various other functional forms. It can be used to consider the multiplicity of scale and technical change combination.

The main purpose of this research is to use the GBC formulation for productivity analysis of Pakistani Textile Industry (large scale). Productivity growth will be analyzed by considering input price effects, non-neutral scale effects and biased technical change. For estimating the textile industry a four input model will be used. The inputs will be capital, labor, energy and intermediate material. The model estimation is based on the maximum likelihood method, and the main hypotheses tested are neutrality of technical change and homothecity.

In Chapter II the general framework of the model will be presented. In particular the technical change and scale effects parameters will be given due consideration, including total factor productivity.

Chapter III will be devoted to empirical implementation and specification of the stochastic frame work for share and cost equations. The advantage of stochastic specification is that the maximum likelihood function is continuous at the value $\lambda = 0$, the transformation parameter. The test criteria will also be considered in this chapter.

In Chapter IV the GBC functional form will be utilized to estimate a four input model of the Pakistani Textile Industry. For estimation, the data used will be from the Census of Manufacturing Industries, Pakistani Statistical Yearbook and Economic Survey of Pakistan. The important features of the data and some relevant empirical evidence will be considered. The focus of the estimated results will be the choice of appropriate technology and functional form, cost function properties and various elasticities estimates, returns to scale and total factor productivity and their responses to changes in the explanatory variables—input prices, output and time.

In the last chapter the summary and conclusions based on the analysis will be presented to facilitate the decision maker/policy maker's decisions for future courses of action.

CHAPTER II. ECONOMETRIC INVESTIGATION OF THE MODEL

The fundamental assumption is that of relationships between outputs and inputs, which can be represented in some mathematical form as

$$Y(z)=0 \quad \text{Equation 2.1}$$

where z is a real valued, m -dimensional vector containing both inputs and outputs produced in a given time period. If we explicitly show the inputs and outputs in the functional form then the above equation can be written as

$$Y(y,x)=0 \quad \text{Equation 2.2}$$

where x is an n -dimension vector of non-negative inputs and y is an $(m - n)$ -dimensional vector of non-negative outputs. The inputs and outputs included in the function are those on which effective control is possible, are economically scarce and are definitely positive. The Equations 2.1 and 2.2 are very general representations of technology for a multiplicity of outputs and inputs. In our case, we are only concerned with single outputs. Thus 'Y' can be assumed as scalar, and the solution to Equation 2.2 can take the form

$$y=f(x) \quad \text{Equation 2.3}$$

where $f(x)$ is a single valued function of x 's (for a unique combination of inputs x , there corresponds a unique level of output). Therefore, it is always assumed that the production function yields the maximum output for an arbitrary input vector. If this maximum level is not attained, there will be technical inefficiencies.

Properties of Production Function $f(x)$

1. If x' (another input combination) $\geq x$ (existing), then $f(x') \geq f(x)$. This gives monotonic behavior of the production function.
2. If $x' > x$, then $f(x') > f(x) \Rightarrow$ strict monotonicity. These two properties show that marginal productivities are positive, and if $f(x)$ is differentiable then marginal productivity of x_i is

$$MP_i = \partial f(x) / \partial x_i \quad \text{Equation 2.4}$$

3. The input requirement set denoted as $V(y)$ which is the combination of all inputs capable of producing the desired level of y , is convex, i.e., $V(y) = \{x: f(x) \geq y\}$ is a convex set. This is quasi-concavity of $f(x)$, and is implied by concavity of $f(x)$; that is, for any arbitrary value of θ such that

$$0 \leq \theta \leq 1, \quad f(\theta x^0 + (1-\theta)x^*) \geq \theta f(x^0) + (1-\theta)f(x^*) \quad \text{Equation 2.5}$$

Convexity can be defined as if x_1 and x_2 are capable of producing 'y', then any weighted average of these two input bundles also can, i.e., if x_1, x_2 are element of $V(y)$ then $x_3 = \theta x_1 + (1-\theta)x_2$ is also an element of $V(y)$. If $V(y)$ is strictly convex set, then $f(x)$ is said to be strictly quasi-concave. Also Equation 2.5 says that as the utilization of a particular input rises, keeping all others fixed, there will be no marginal increase in the output, that is that the law of diminishing marginal productivity applies. In the case where $f(x)$ is twice continuously derivable, the Equation 2.5 means that the diagonal elements of the Hessian matrix $(\partial^2 f(x) / \partial x_i^2)$ are non-positive and the matrix itself is negative semi-definite.

4. Strictly positive output cannot be produced without any input, i.e., $f(0_n) = 0$, where 0_n is the null vector, which is known as weak essentiality. Also all inputs are essential to the production of positive amount of output, i.e., $f(x_1, x_2 - x_i, 0 \dots x_{i+1} \dots x_n) = 0$ for all x_i and is known as strict essentiality.
5. It is always possible to produce any positive output, i.e., $V(y)$ is a closed and non-empty set for $y > 0$ closedness means that $f(x)$ has not discontinuities over the boundaries and has well defined constrained maxima and minima.
6. The function $f(x)$ is finite, non-negative, real valued, and single valued for all non-negative and finite x .

Definitions

The function $f(x)$ is continuous, twice-derivable, non-decreasing and quasi concave with input vector $\underline{x} \geq 0$ and output y . The following definitions apply to $y = f(\underline{x})$

Average Product: The average product of x_i is given as

$$AP_i = f(x)/x_i = y/x_i \quad \text{Equation 2.6}$$

Marginal Product: It is the change in output associated with small change of x_i or

$$MP_i = \partial f(x)/\partial x_i = \partial y/\partial x_i \quad \text{Equation 2.7}$$

Elasticity of Output (ϵ_i): It is the percentage change in output corresponding to 1% change in the input i or

$$\epsilon_i = (\partial f(x)/\partial x_i)/y/x_i = \left(\frac{\partial f(x)/\partial x_i}{f(x)/x_i} \right) = MP_i/AP_i = \frac{\partial \ln f(x)}{\partial \ln x_i} \quad \text{Equation 2.8}$$

When $\epsilon_i = 1$, $MP_i = AP_i$ which implies that the average product is maximized.

Elasticity of Scale: It is also known as function coefficient or passuss coefficient.

It gives the scalar valued measure of the changes in output corresponding to simultaneous changes in all inputs. Mathematically the elasticity of scale ϵ is defined as

$$\epsilon = \left. \frac{\partial \ln f(\lambda x)}{\partial \ln \lambda} \right|_{\lambda=1} \quad \text{Equation 2.9}$$

and shows how output changes as each x is multiplied with a scalar λ . There are four important facts related to elasticity of scale, by which the production can be characterized.

(a) Constant returns to scale If $\epsilon=1$, then the production function exhibits constant returns to scale and, for any x , $f(\lambda x)=\lambda f(x)$, which means that multiplying all inputs by a scalar is same as multiplying the output by the same scalar. Also for $\epsilon=1$, the isoquants are equally spaced.

(b) Decreasing returns to scale When $\epsilon < 1$, the function exhibit decreasing return to scale and isoquants spread out more and more if one move along a ray away from the origin. Also

$$f(\lambda x) \leq \lambda f(x) \text{ for } \lambda \geq 1 \quad \text{Equation 2.10}$$

The difference between diminishing marginal productivity and decreasing returns to scale is that the first one is the measure of output variation in response to single input change, while the latter is associated with simultaneous changes in all inputs.

(c) Increasing returns to scale If $\epsilon > 1$, then function shows increasing return to scale and isoquants spread out as one moves along a line towards origin. Also $f(\lambda x) \geq \lambda f(x)$ for $\lambda > 1$. Equation 2.11

(d) Economies of scales In the case of decreasing return to scale case it is preferable to build 'm' small plants than one big one, provided the cost to build is the same in both cases, while for increasing return the bigger plant will take over. With constant returns it doesn't matter which plant is built. Hence we say that there will be diseconomies of scale in the first case, and economies of scale in the second case. When $\epsilon < 1$ or $\epsilon > 1$ the entrepreneur is concerned with the economies or diseconomies of scale, while at $\epsilon = 1$ he is indifferent. It is also worth mentioning that ϵ is an instantaneous local measure and it may change its values over the entire feasible input space to constant, increasing or decreasing returns. Evaluating Equation 2.9 we get that

$$\epsilon = \left. \frac{\partial \ln f(\lambda x)}{\partial \ln \lambda} \right|_{\lambda=1} = \sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} \right) \left(\frac{x_i}{y} \right) = \sum \epsilon_i = \sum_{i=1}^n MP_i / AP_i \quad \text{Equation 2.12}$$

From Equation 2.12,

$$\epsilon < 1 \Rightarrow \sum \epsilon_i < 1 \Rightarrow \sum \frac{MP_i}{AP_i} < 1 \Rightarrow \sum MP < \sum AP \quad \text{Equation 2.13}$$

It shows that decreasing returns to scale functions must have all their marginal product less than the corresponding average product. This also shows that for a production function with decreasing returns to scale over the entire input space, the marginal contribution of an input to an output will always be less than its average contribution. Output will increase at a slower rate than input.

Transform of a Production Function

A transform of production function $f(x)$ is defined as $F(f(x))$, where $F(\cdot)$ is a twice-derivable, finite, non-negative and non-decreasing function of $f(x)$ and preserves all the basic properties of $f(x)$. The idea behind this is that once a valid function is specified, a family of the functions can be generated by the transform $F(f(x))$.

Homothetic Functions

A function is said to be homothetic if it can be expressed as $y = F(f(x))$, where F is a monotonic increasing function of a single variable and $f(x)$ is linearly homogenous function of x , that is $f(\lambda x) = \lambda f(x)$ or $f(x)$ is homogenous of degree 1. Thus homogeneity of degree 1 is a special case of homotheticity of a function. Homothetic production functions are particularly important because this is the only class of transforms in which proportional changes in all inputs are accurately reflected by the same proportional change in aggregate input. If we differentiate the above function with respect x_i and x_j then we get

$$\frac{\partial y / \partial x_i}{\partial y / \partial x_j} = \frac{\partial f(x) / \partial x_i}{\partial f(x) / \partial x_j} = \frac{MP_i}{MP_j} = \text{Marginal rate of technical substitution between input } i \text{ and input } j \text{ or } MRTS_{ij} \quad \text{Equation 2.14}$$

As $f(x)$ is linearly homogenous, each derivative on RHS is homogenous of degree 0.

From $f(\lambda x) = \lambda f(x)$ we get by differentiation:

$$\frac{\partial f(\lambda x)}{\partial x} \cdot \lambda = \lambda \frac{\partial f(x)}{\partial x} \text{ or } \partial f(\lambda x) / \partial x = \partial f(x) / \partial x \quad \text{Equation 2.15}$$

This gives us the homogeneity of degree 0. For a function to be homothetic, the first partial derivatives, if they exist, must be homogenous of degree 0. In Equation 2.14, if

we multiply the argument of the function in the numerator and the denominator by a scalar, it will not alter the value of the equation. This means that the ratio of the MP_i/MP_j remains constant or $MRTS_{ij}$ is constant along a ray from the origin. When the $MRTS$ is constant, the shifts in the isoquants will be parallel. Hence, homothecity ensures that all isoquants are parallel and have the same slope.

Separability

If the stages of an aggregate production function can be broken down such that each stage is a production function, then this is separable technology. Mathematically, the marginal rate of technical substitution between inputs defines separability for a twice-derivable and continuous production function. Let us see how inputs x_i and x_j are separable from input x_k , when changes in x_j are affecting x_i in response to the changes in x_k . We have $MRTS_{ij} = - MP_i/MP_j$, or

$$MRTS_{ij} = -\frac{\partial f / \partial x_i}{\partial f / \partial x_j}, \text{ because we know that } MP_i = \partial f / \partial x_i, MP_j = \partial f / \partial x_j \quad \text{Equation 2.16}$$

Differentiating Equation 2.16 with respect to x_k :

$$\frac{\partial(MRTS_{ij})}{\partial x_k} = - \frac{\partial}{\partial x_k} \left(\frac{\partial f / \partial x_i}{\partial f / \partial x_j} \right) \quad \text{Equation 2.17}$$

If this last is zero, then x_i and x_j are separable from x_k , otherwise they are not. This tells us that the slope of the isoquant in i - j space is constant, irrespective of the k space. Also i and j are not equal to k . Another form of Equation 2.17 is:

$$\begin{aligned}
\frac{\partial(MRTS_{ij})}{\partial x_k} &= - \frac{\partial(MP_i/M P_j)}{\partial x_k} \frac{\partial[(M P_i)(M P_j^{-1})]}{\partial x_k} \\
&= - M P_i (M P_j)^{-2} \frac{\partial M P_j}{\partial x_k} + (M P_j)^{-1} \frac{\partial M P_i}{\partial x_k} \Rightarrow - f f_{jk} / f_j^2 + f_{ik} / f_i = 0 \Rightarrow \frac{f_{jk}}{f_j^2} = \frac{f_{ik}}{f_i^2}, \\
\text{where } f_{jk} &= \frac{\partial}{\partial x_k} (\partial f / \partial x_j) = \partial^2 f / \partial x_k \partial x_j,
\end{aligned}$$

and multiplying both sides by x_k the logarithmic form is

$$\partial \ln f_j / \partial \ln x_k = \partial \ln f_i / \partial \ln x_k \text{ where } f_i = \partial f / \partial x_i, f_j = \partial f / \partial x_j \quad \text{Equation 2.18}$$

Equation 2.18 shows us that under separability, the elasticity of the marginal product of x_i w.r.t. x_k equals the elasticity of the marginal product of x_j w.r.t. x_k . Generally, if the input vector I is partitioned into subsets such that $\hat{I} = (I^1, I^2, I^r, I^k, \dots, I^m)$ where \hat{I} corresponds to $\hat{x} = [x^1 \dots x^n]$, then the production function is weakly separable in the partition \hat{I} if

$$\frac{\partial}{\partial x_k} \left(\frac{\partial f(x) / \partial x_i}{\partial f(x) / \partial x_j} \right) = 0, \quad i, j \in I^r, k \in I^r \quad \text{Equation 2.19}$$

$f(x)$ is weakly separable in the partition \hat{I} , if the $MRTS_{ij}$ in the same subset, are independent of all inputs that are not the elements of their corresponding subset.

The function $f(x)$ is supposed to be strongly separable in partition \hat{I} if

$$\frac{\partial}{\partial x_k} \left(\frac{\partial f(x) / \partial x_i}{\partial f(x) / \partial x_j} \right) = 0, \quad i \in I^r, j \in I^v, k \in I^r \cup I^v \quad \text{Equation 2.20}$$

(see Figure 2.1). It shows that a strongly separable function can be written as $y = F(f^i(x^i))$ where in $\Sigma f^i(x^i)$, i = stages of production. This is because $y = F(f(x))$ is a homothetic function and $f(x)$ is the aggregate of $f^i(x^i)$ and is linearly homogeneous. Thus a strongly separable function is also a homothetic function. The function is factor wise separable if

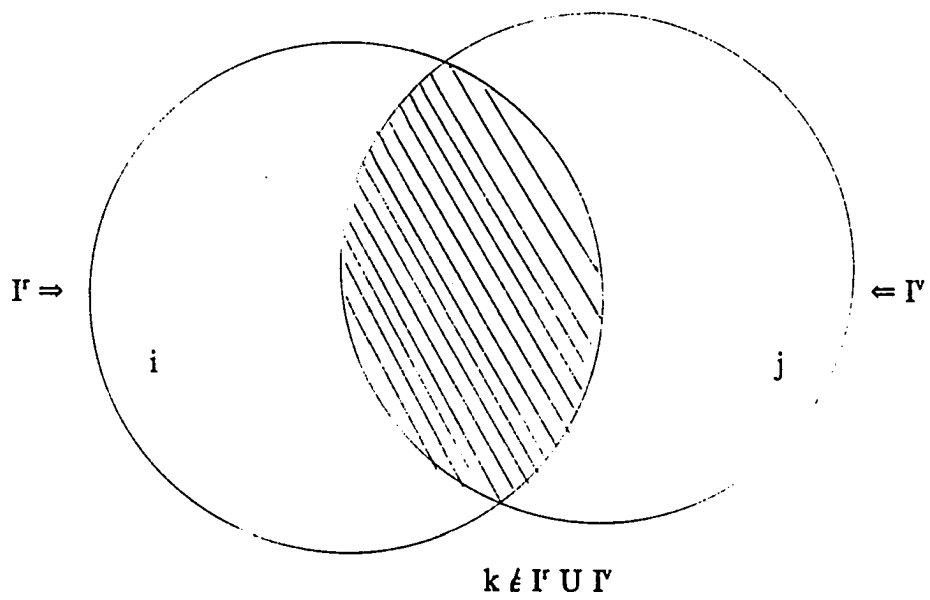


Figure 2.1 Explanation of strong separability

for x_i , x_j and x_k as mentioned above

$$\frac{\partial}{\partial x_k} \left(\frac{\partial f / \partial x_i}{\partial f / \partial x_j} \right) = 0, \quad k \neq i, \quad k \neq j \quad \text{Equation 2.21}$$

The assumption of separability imposes restrictions on the function and thus decreases the number of candidate production functions. It also greatly reduces the number of parameters to be estimated.

Cost Functions

For profit maximizing firms, production and cost functions are integral parts of one another because the profit can be maximized either maximizing output for a given level of cost involved or minimizing cost for given level of output. Initiatively it looks easier to cut down the cost for constant output than to increase output, without increasing the cost. The maximization of output or minimization of cost means maximization of the profit π . The cost function is defined as "the minimum cost of producing a given level in a given time period with given input prices." Mathematically this can be written as

$$C = C(w, y) = \min_{x \geq 0} [w \cdot x; x \in V(y)], \quad \text{Equation 2.22}$$

where w is a strictly positive input price vector and x is the vector of inputs. $V(y)$ is the input requirement set. The cost function or curve is the locus of the set of all possible pairs of output and cost along the expansion path as shown in Figure 2.2. It can be seen easily from the diagram that cost is constant along the isocost curves for given input combinations, while isoquants (the locus of input combinations for which output is fixed) gives the maximum output for given cost is the point where cost line become tange to isoquant. Also cost can be minimized for given output where an isoquant is tangent to

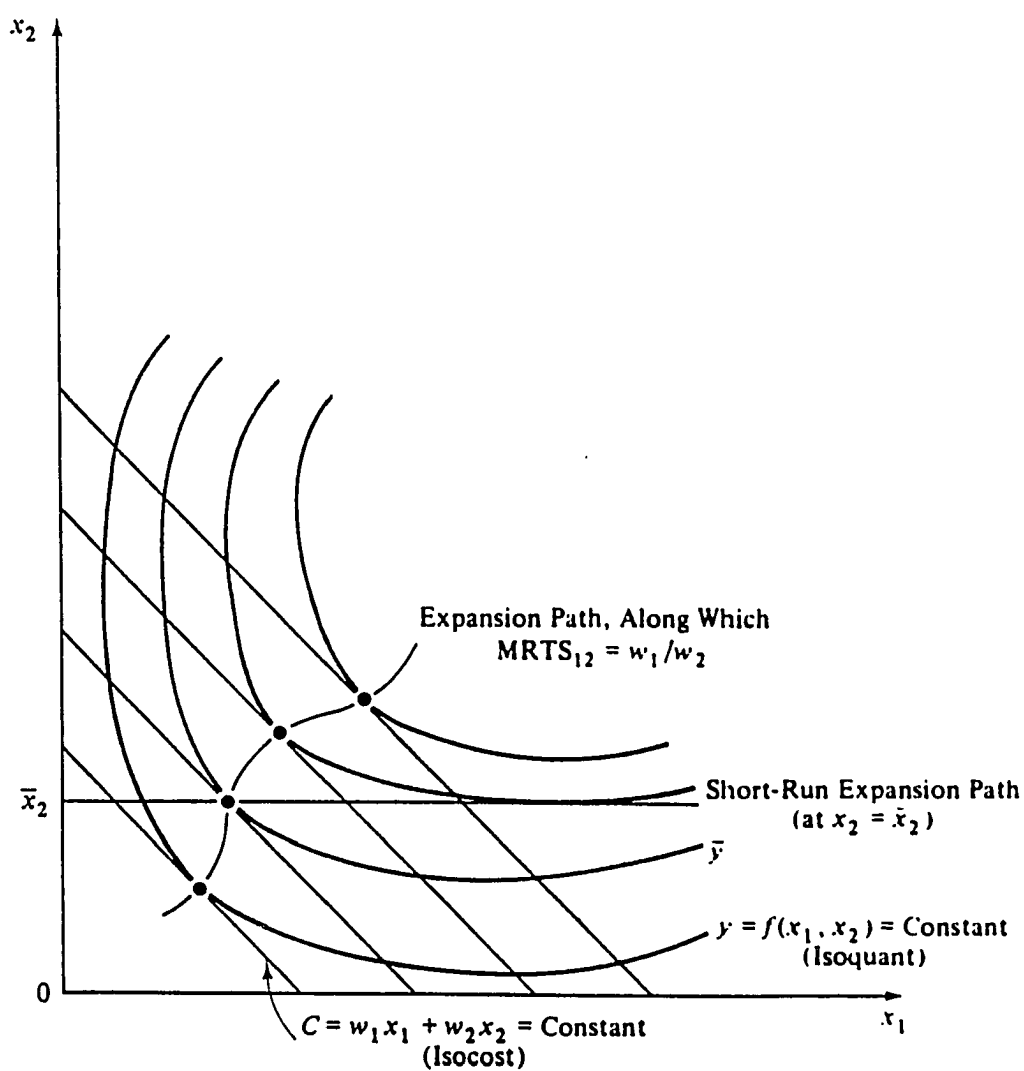


Figure 2.2 Expansion paths for the firm (Intriligator, 1978, p. 253)

the cost line. The expansion path is the only maximum output or minimum cost solution under equilibrium condition for a firm, and is the locus of tangency points of the isocost and the isoquant curves.

If isoquants are characterized by the production function $y=f(x)$, then the profit π of the firm to be maximized is given by revenue minus costs for a given output y or mathematically

$$\pi = py - C = pf(x) - C(w,y) \quad \text{Equation 2.23}$$

where p equal output price, or

$$\pi = pf(x) - w_i x_i \quad \text{Equation 2.24}$$

For maximum π first partial derivatives should exist and equal to zero, or which shows

$$\frac{\partial \pi}{\partial x_i} = p \frac{\partial f}{\partial x_i} - w_i = 0 \Rightarrow \frac{\partial f}{\partial x_i} = \frac{w_i}{p} = MP_i, \quad \text{Equation 2.25}$$

that for maximum profit π , the marginal product of each input i must be equal to its real wage, the wage input price divided by price of the output. As we know that marginal rate of technical substitution is the ratio of their marginal products, therefore,

$$MRTS_{ij} = MP_i / MP_j = \frac{w_i/p}{w_j/p} = w_i/w_j \quad \text{Equation 2.28}$$

In the case of two inputs x_1, x_2 the cost function

$$C = w_1 x_1 + w_2 x_2 = \text{constant, for an isocost curve.} \quad \text{Equation 2.29}$$

Differentiating C totally we get

$$w_1 dx_1 + w_2 dx_2 = 0 \quad \text{or} \quad dx_2/dx_1 = - w_1/w_2 \quad \text{Equation 2.30}$$

Equation 2.30 gives the slope of the isocost curve.

Also if we differentiate $y = f(x) = f(x_1, x_2)$ totally for given (constant) level of output we get that

5. An increasing output cannot decrease the cost, i.e., if $y \geq y'$, then $c(w, y) \geq c(w, y')$.
6. It is costless to produce nothing, i.e., zero fixed cost, i.e., $c(w, 0) = 0$.
7. For differentiable cost function, the optimal (cost-minimizing) factor input demand is given by the slope of the function, i.e.,

$$c(w, y) = \min_{x \geq 0} c = \sum_{i=1}^n w_i x_i. \quad \text{Differentiating w.r.t } w_i \text{ we get} \quad \text{Equation 2.32}$$

$$\partial c / \partial w_i = x_i \quad \text{or} \quad x_i = \partial c(w, y) / \partial w_i$$

Equation 2.32 is also known as Shephard's lemma. This shows that behavior of derived demand is determined by the properties of cost function. It holds strictly for the convex set of input requirements $[V(y)]$.

Derived Demand Behavior and Input Price Changes

From Equation 2.32 we know that

$$x_i \text{ (derived demand input vector)} = \partial c(w, y) / \partial w_i$$

The effect of a price increase of w_j is given by differentiating x_i with respect to w_j partially

$$\partial x_i / \partial w_j = \partial^2 c / \partial w_i \partial w_j \quad \text{Equation 2.33}$$

The right hand side of Equation 2.33 is the Hessian matrix of the cost function. Hence the derived demand responses to input price changes can be calculated from the Hessian matrix. Applying Euler's theorem to the homogeneous derived demand equation gives us by differentiation for each w_j

$$\sum_j \frac{\partial x_i}{\partial w_j} \cdot w_j = 0, \text{ the homogeneity conditions.} \quad \text{Equation 2.34}$$

Derivability and concavity of the cost function $c(w,y)$ implies that the Hessian matrix is negative semidefinite:

$$\partial x_i / \partial w_j \leq 0 \quad \text{Equation 2.35}$$

Also, a symmetry or reciprocity condition can be obtained as

$$\partial x_i / \partial w_j = \partial^2 c / \partial w_i \partial w_j = \partial^2 c / \partial w_j \partial w_i = \partial x_j / \partial w_i. \quad \text{Equation 2.36}$$

The derived demand elasticity can be given as

$$\epsilon_{ij} = \frac{\partial x_i / \partial w_j}{x_i / w_j} = \frac{\partial \ln x_i}{\partial \ln w_j}. \quad \text{Equation 2.37}$$

But from Equation 2.34 $\sum \epsilon_{ij} = 0$ and from Equation 2.35 $\epsilon_{ii} \leq 0$.

Derived demand elasticities are not symmetric, unless normalized properly, i.e., $\epsilon_{ij} \neq \epsilon_{ji}$. However,

$$\epsilon_{ij} S_i = \epsilon_{ji} S_j, \text{ where } S = wx/c(w,y), \quad \text{Equation 2.38}$$

an expression described as cost share.

Marginal cost (MC): Defined as the slope of the total cost curve, or mathematically

$$MC = \frac{\partial c}{\partial y} \geq 0, \text{ because } y \text{ is non-decreasing, or}$$

$$MC = \frac{\partial c(tw,y)}{\partial y} = t \frac{\partial c(w,y)}{\partial y}, \quad t > 0, \quad \text{Equation 2.39}$$

which shows that MC is linearly homogeneous in the input prices.

If we differentiate MC with respect to each w_i we get

$$\frac{\partial MC}{\partial w_i} = \sum \frac{\partial^2 c(w,y)}{\partial y \partial w_i} w_i = \sum_i \frac{\partial^2 c(w,y)}{\partial w_i \partial y} w_i = \frac{\partial}{\partial y} \sum_i \frac{\partial c(w,y)}{\partial w_i} w_i \quad \text{Equation 2.40}$$

$$\frac{\partial}{\partial y} \left(\sum_i w_i x_i \right) = \frac{\partial c(w,y)}{\partial y} \quad \text{Equation 2.41}$$

which shows that equiproportional changes in all input prices will shift the MC curve in a parallel form.

Cost and Changes in Output

The response of derived demand input to changes in output is given by differentiating the derived demand Equation 2.32 with respect to output y :

$$\begin{aligned} x_i &= \frac{\partial c}{\partial w_i} \\ \frac{\partial x_i}{\partial y} &= \frac{\partial^2 c}{\partial w_i \partial y} = \frac{\partial^2 c}{\partial y \partial w_i} = \frac{\partial}{\partial w_i} \left(\frac{\partial c}{\partial y} \right) = \frac{\partial}{\partial w_i} (MC) \\ \text{or } \frac{\partial x_i}{\partial y} &= \frac{\partial MC}{\partial w_i} \end{aligned} \quad \text{Equation 2.42}$$

It shows that the i th input response to a unit change in output is equal to the change in MC with respect to a unit change in input price.

If $\partial x_i / \partial y < 0$, then x_i is an inferior or regressive input.

If $\partial x_i / \partial y > 0$, then x_i is a normal or progressive input.

Average cost It is defined as total cost divided by total output. The ratio of marginal cost to average cost is known as the cost elasticity (n) w.r.t. output or cost flexibility.

$$n(w,y) = \left[\frac{MC}{AC} \right] = \left[\frac{\partial c / \partial y}{c/y} \right] = \left[\frac{\partial \ln c}{\partial \ln y} \right] \quad \text{Equation 2.43}$$

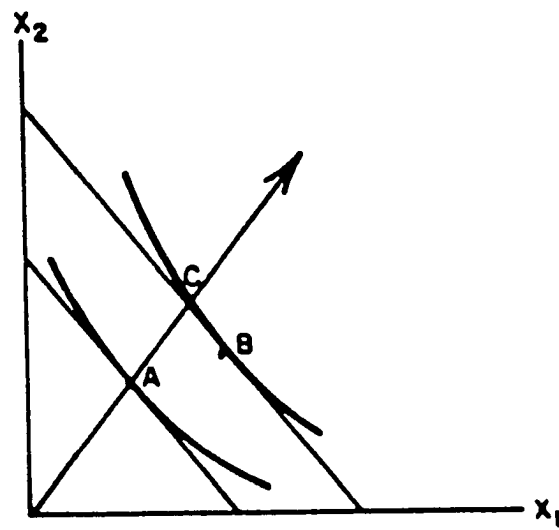


Figure 2.3 Elasticities of scale and size (Chambers, 1990, p. 73)

parallel shift of isoquants, or homothetic shift. Such a shift implies that the production function is homothetic, and coincidence of points C and B will suggest a homothetic cost function, with the property of equality of (ϵ) and (ϵ^*) . This gives us that the elasticity of scale (ϵ) , and hence of size (ϵ^*) , for homothetic function, is only a function of the output level and is independent of x_i .

Therefore for homothetic functions we can write $h(y) = f^*(x)$, and

$$\begin{aligned}
 c(w,y) &= \min_{x>0} [w \cdot x: h(y) \leq f^*(x)], \text{ dividing by } h(y) \text{ we have} \\
 &= \min_{x>0} [w \cdot x: 1 \leq f^*(x)/h(y)], \text{ and letting } x=zh(y), \\
 &= \min_{z>0} [h(y)z \cdot w: 1 \leq f^*(z)] \\
 &= h(y) \min_{z>0} [z \cdot w: 1 \leq f^*(z)] = h(y) c(w) \\
 \text{or } c(w,y) &= h(y) c(w)
 \end{aligned}$$

Equation 2.44

There are three important facets of homothetic technologies

- (i) The economies of scale are measured easily and comprehensively.
- (ii) The cost function can easily be traced back to the production function

with homotheticity of the production function.

(iii) Separability of output from input prices assures a homothetic cost function consistent with the production function.

As we discussed earlier, homogeneous functions are also homothetic functions.

Therefore, if $f(x)$ is homogenous of degree k then we can write

$$f(x) = [g(x)]^k, \text{ where } g(x) \text{ is linearly homogeneous.}$$

Equation 2.45

If $y=f(x)$ then

$$y=[g(x)]^k \quad \text{or} \quad [y]^{1/k} = g(x) \quad \text{Equation 2.46}$$

and the cost function (c.f. Equation 2.44) is

$$c(w,y) = [y]^{1/k} c(w) = h(y) c(w) \quad \text{Equation 2.47}$$

When cost flexibility (n) is one, the average cost reaches the value of the marginal cost, at the point at which the slope of a tangent to the average cost curve becomes zero, and the average cost increases to the right and decreases to the left of this point, indicating that the cost function must be convex at that point (see Figure 2.4). As to ϵ^* , being the reciprocal of n and hence equal to one, it gives us constant return to size at that point. To the left of the point, we have decreasing return to size ($\epsilon^* < 1$) with increasing AC, and, to the right, increasing return to size ($\epsilon^* > 1$) with decreasing AC. This shows that when the cost function is convex, the production function is concave, which gives us a sufficiency condition (see Figure 2.4).

Duality of Cost and Production Functions

One of the most important discoveries of linear programming is the concept of duality and its important ramifications. According to duality every linear problem has associated with it another problem, called the "dual". The relationship between the dual problem and the original problem (called Primal) proves to be extremely useful in a variety of ways and especially in the economic interpretation of production functions in terms of cost functions. One application involves an attempt to isolate circumstances in which technology properties can be derived from the economic behavior of optimizing agents. The use of cost functions to describe technology involves the equivalence of the

specification of a well-behaved cost function and the specification of a well-behaved production function. According to McFadden (1978a), the cost function is a "sufficient statistic", because all the economically relevant information about technology can be obtained from the cost function.

Production Function (Primal Problem)

A rational producer in a competitive market will either set output for given cost of factor inputs to maximize his profit or minimize the cost of production at given output and factor inputs prices.

The problem of maximizing output at given input factor prices can be written in mathematical form as

$$\begin{aligned} \text{Max } y &= \sum_{j=1}^n p_j y_j \\ \text{such that } \sum_{j=1}^n a_{ij} y_j &\leq w_i \quad i = 1, 2, \dots, m \\ y_1, y_2, \dots, y_n &\geq 0 \end{aligned} \quad \text{Equation 2.48}$$

where y is the objective function to be maximized

y_j are decision variables $y = f(x_1, \dots, x_n)$

a_{ij} , w_i , and p_i are parameters of the model.

In vector notation this can be written as

$$\begin{aligned} \text{MAX } y &= py \\ \text{S.T. } Ay &= w \quad (w \geq 0), y \geq 0 \end{aligned} \quad \text{Equation 2.49}$$

where y is the output vector.

The generalization of Shephard's (1953) results by Uzawa (1962) shows that the cost function can be used to reconstruct the input requirement set (the production

function) from which it was generated. As was mentioned above, a producer will maximize output or minimize cost, and both points of view lead to the point of tangency of the factor price frontier (isocost) with the isoquant (see Figure 2.5). From the factor price frontier the slope of the isoquant and relative input utilization can be obtained.

To see how the price of an input i is affected by changes in other input prices for cost to be constant, we can solve for w_i in terms of other input prices as

$$w_i = w_i(w_1 \dots w_{i-1} \dots w_n) \quad \text{Equation 2.50}$$

The constant cost function is $c(w,y) = 1$; differentiating w.r.t w_j

$$\frac{\partial w_i}{\partial w_j} = \frac{\partial c(w,y)/\partial w_j}{\partial c(w,y)/\partial w_i} \quad \text{Equation 2.51}$$

is the slope of the factor price frontier (isocost) in i - j space (see Figure 2.6). We also know that $\partial c(w,y)/\partial w_j = x_j$, and

$$\frac{\partial c(w,y)}{\partial w_i} = x_i \therefore \frac{\partial w_i}{\partial w_j} = -x_j/x_i, \quad \text{Equation 2.52}$$

illustrating the fact that a price increase of one input will lead to a decrease in the price of another to maintain $c(w,y)=1$.

In the case of cost minimization, the ratio of marginal productivities is equal to the negative of the slope of the isoquant as

$$\frac{\partial x_i}{\partial x_j} = -w_j/w_i \quad \text{Equation 2.53}$$

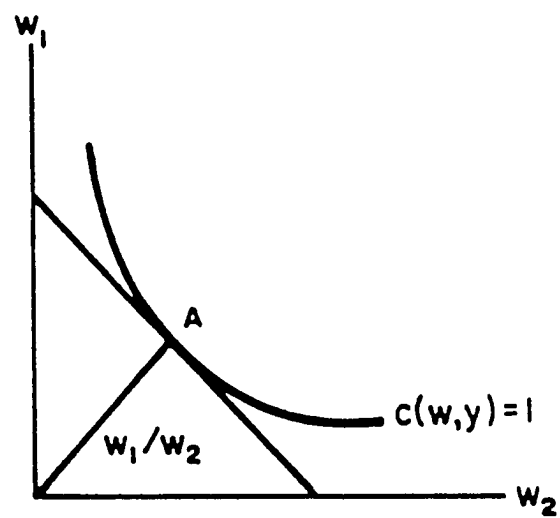


Figure 2.5 Dual relationship between factor price frontier and isoquant (Chambers, 1990, p. 90)

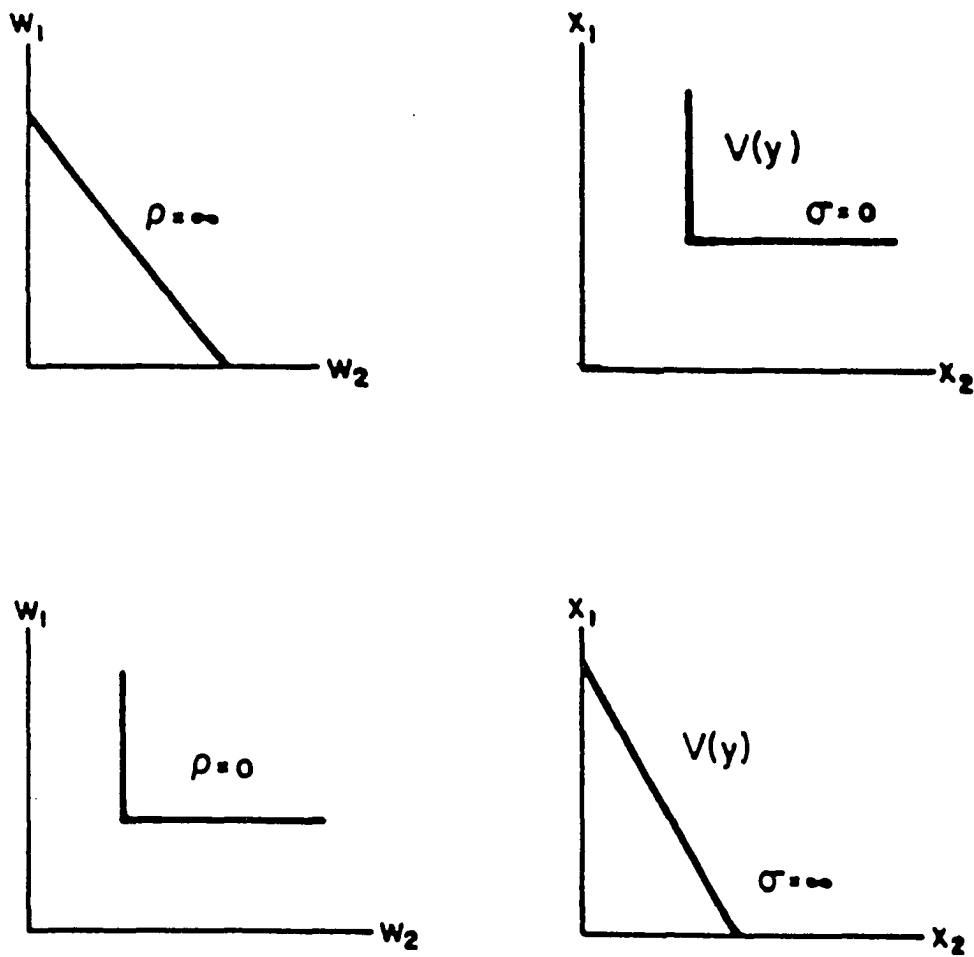


Figure 2.6 Factor price frontiers and isoquant (Chambers, 1990, p. 91)

From Equation 2.52 and Equation 2.53, it is clear that from the form of the cost function information, the production function (isoquant) can be traced back. This is a consequence of the duality theorem, which says that, for every original programming problem, there is a dual problem for which the optimal solution will be exactly the solution to the original primal problem. Also the dual of this dual problem is the original primal problem.

Let us use 2.48 as the original primal problem, for which we will write the dual problem as

$$\begin{aligned} \text{Minimize } C &= \sum_{i=1}^m w_i x_i \\ \text{such that } \sum_{i=1}^m a_{ij} x_i &\geq p_j \quad j = 1, 2, \dots, n, \quad x_i \geq 0, \quad i = 1, 2, \dots, m \end{aligned} \quad \text{Equation 2.54}$$

or vectorically

$$\begin{aligned} \text{Min } C &= wx \\ \text{such that } ax &= p \quad (p \geq 0) \quad x \geq 0 \end{aligned}$$

From 2.48 and Equation 2.54 we clearly see that the dual variable corresponds to the primal constraints and the dual constraints correspond to the primal variables. There is a direct correspondence between the optimal solution of the primal and the dual. The two optimal solutions yield the same value for their respective objective functions, or, mathematically, if $(y_1^*, y_2^* \dots y_n^*)$ and $(x_1^*, x_2^* \dots x_m^*)$ are optimal solution for the primal and the dual respectively, then

$$\sum_{j=1}^n p_j y_j^* = \sum_{i=1}^m w_i x_i^*,$$

which is fundamental theorem of duality.

In Figure 2.5, the elasticity of the curvature of the isocost or factor price frontier can be defined as

$$\begin{aligned} \rho &= \frac{d \ln(w_1/w_2)}{d \ln(c_2/c_1)}, \text{ but } \frac{\partial c(w,y)/\partial w_2}{\partial c(w,y)/\partial w_1} = \frac{x_2}{x_1} = \frac{c_2}{c_1}, \\ \therefore \rho &= \frac{d \ln(w_1/w_2)}{d \ln(x_2/x_1)} \end{aligned} \quad \text{Equation 2.55}$$

The elasticity of the curvature of the isoquant is given

$$\begin{aligned} \sigma &= \frac{d \ln(x_2/x_1)}{d \ln(f_1/f_2)}, \quad f_1 = \frac{\partial f}{\partial x_1}, \quad f_2 = \frac{\partial f}{\partial x_2}, \\ \text{but } \frac{\partial f}{\partial x_1} &= MP_1, \quad \frac{\partial f}{\partial x_2} = MP_2 \therefore \frac{f_1}{f_2} = \frac{MP_1}{MP_2} = -\frac{w_1}{w_2} = MRTS \\ \therefore \sigma &= \frac{d \ln(x_2/x_1)}{d \ln(w_1/w_2)} \end{aligned} \quad \text{Equation 2.56}$$

Hence $\rho = 1/\sigma$ which shows that the curvature of the factor price frontier is integrally linked to the curvature of the isoquant. If $\rho = \infty$, then $\sigma = 0$ and when $\sigma = \infty$, then $\rho = 0$. This effect is shown in Figure 2.6 for a factor price frontier and an isoquant.

For a production function which is twice-derivable, non-decreasing and quasi-concave, and for which the vector formulation is $y=f(x)$, with x the vector of non-negative inputs, we can write a cost-function dual to the production function that will minimize cost of production subject to given output as

$$c = c(w,y).$$

Here $c(w,y)$ is a non-decreasing linearly homogeneous and convex function of strictly positive input prices. $w > 0$.

Transformations

The main purpose of transformations of both independent (x) and dependent (y) variables is to insure the simplest possible regression model in term of transformed variables, and to assure, in the case of multiple regression, and, in particular, in the analysis of response surfaces, that the following assumptions hold:

- (i) simplicity of structure for $E(Y)$ or $E(X)$;
- (ii) constancy of error variance;
- (iii) normality of distributions;
- (iv) independence of observations.

If the assumptions (i) through (iii) are not satisfied in terms of the original model non-linear transformation of y or x may improve matters. The Cox-Box transformation with parameter λ can be used to define a transformation. There are two ways of transforming the dependent variable:

$$y^\lambda = \begin{cases} \frac{y^\lambda - 1}{\lambda} & \text{if } \lambda \neq 0 \\ \ln y & \text{if } \lambda = 0 \end{cases} \quad \text{provided } y > 0 \text{ or,} \quad \text{Equation 2.57}$$

if an unidentified origin λ_2 is included, then

$$y^\lambda = \begin{cases} \frac{(y+\lambda_2)^{\lambda_1} - 1}{\lambda_1} & \text{if } \lambda_1 \neq 0 \\ \ln(y+\lambda_2) & \text{if } \lambda_1 = 0 \end{cases} \quad \text{provided } y > -\lambda_2. \quad \text{Equation 2.58}$$

As an analysis of variance is not changed by linear transformation, Equation 2.57 can be written as

$$y^\lambda = \begin{bmatrix} y^\lambda & \lambda \neq 0 \\ \ln y & \lambda = 0 \end{bmatrix} \quad \text{Equation 2.59}$$

but Equation 2.57 is preferable for theoretical analysis because it is continuous at $\lambda=0$.

Let $F(x)$ be an arbitrary function that can be represented in second-order numerical approximation in terms of its arguments. If we evaluate at $x^\circ=(1,1,\dots,1)$ we obtain:

$$\begin{aligned} F(x) &= F(x^\circ) + \sum_{i=1}^n \frac{\partial F(x^\circ)}{\partial x_i} x_i + \frac{1}{2} \sum_i^n \sum_j^n \frac{\partial^2 F(x^\circ)}{\partial x_i \partial x_j} x_i x_j \\ F(x) &= \gamma_o + \sum_{i=1}^n \gamma_i x_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} x_i x_j \\ \text{where } \gamma_o &= F(x^\circ), \gamma_i = \frac{\partial F(x^\circ)}{\partial x_i}, \beta_{ij} = \frac{\partial^2 F(x^\circ)}{\partial x_i \partial x_j} \end{aligned} \quad \text{Equation 2.60}$$

From the above discussion we can write a production function comprising independent and dependent variables as follows:

$$y = F(x) \quad \text{Equation 2.61}$$

when $\lambda \neq 0$ we can write Equation 2.61 based on Equation 2.57 as

$$\frac{y^\lambda - 1}{\lambda} = F(x). \quad \text{Equation 2.62}$$

Where the second order numerical approximation of $F(x)$ is

$$F(x) = \gamma_o + \sum_{i=1}^n \gamma_i x_i(\lambda) + \frac{1}{2} \sum_i^n \sum_j^n \beta_{ij} x_i x_j \quad \text{Equation 2.63}$$

and

$$x_i(\lambda) = \begin{cases} \frac{x_i^{\lambda/2} - 1}{\lambda/2} & \text{when } \lambda \neq 0 \\ \ln x_i & \text{when } \lambda \rightarrow 0 \end{cases}, \quad \text{Equation 2.64}$$

where y =output and x_i = quantity of i th input.

Equation 2.62 can be written in alternate form as

$$y = [1 + \lambda F(x)]^{1/\lambda} \text{ known as a generalized Cox-Box function.} \quad \text{Equation 2.65}$$

Based on the duality theorem, there will be a cost function corresponding to Equation 2.61, denoted as

$$c = c(w, y), \quad \text{Equation 2.66}$$

where $c(w, y)$ is a non-decreasing, linearly homogeneous, and concave function of w for each y .

Corresponding to the transformation of Equation 2.65, the generalized Cox-Box cost function can be written as

$$c(w, y) = [1 + \lambda G(w)]^{1/\lambda} y \quad \text{Equation 2.67}$$

$$\text{where } G(w) = \gamma_o + \sum_i^n \gamma_i w_i(\lambda) + \frac{1}{2} \sum_i^n \sum_j^n \beta_{ij} w_i(\lambda) w_j(\lambda) \quad \text{Equation 2.68}$$

a second order differential approximation for the cost function $G(w)$, where

$$w_i(\lambda) = \begin{cases} (w_i^{\lambda/2} - 1)/\lambda/2 & \text{when } \lambda \neq 0 \\ \ln w_i & \text{when } \lambda = 0, \end{cases} \quad \text{Equation 2.69}$$

and w_i = price of the input i , c = total cost, and y = output.

As mentioned above for a twice-derivable function Young's theorem says that

$$\frac{\partial^2 F}{\partial x_i \partial x_j} = \frac{\partial^2 F}{\partial x_j \partial x_i} \text{ or } f_{12} = f_{21},$$

or $\beta_{ij} = \beta_{ji} \forall i, j$ and linear homogeneity in prices implies that when factor prices double, the total cost has to double, which gives

$$\sum_i \gamma_i = 1 + \lambda \gamma_o, \quad \sum_j \beta_{ij} = \frac{\lambda}{2} \gamma_i \text{ for } \forall i. \quad \text{Equation 2.70}$$

We can then write Equation 2.67 for the cost function as:

$$\begin{aligned} c(w,y) &= [1 + \lambda G(w)]^{1/\lambda} y, \text{ or} \\ c(w,y) &= \left[1 + \lambda \gamma_o + \lambda \sum_i \gamma_i w_i(\lambda) + \frac{\lambda}{2} \sum_i \sum_j \beta_{ij} w_i(\lambda) w_j(\lambda) \right]^{1/\lambda} y, \text{ or} \\ c(w,y) &= \left[\sum_i \gamma_i + \frac{\lambda}{2} \sum_i \gamma_i w_i(\lambda) + \frac{\lambda}{2} \sum_i \beta_i w_i(\lambda) + \frac{\lambda}{2} \sum_i \sum_j \beta_{ij} w_i(\lambda) w_j(\lambda) \right]^{1/\lambda} y, \text{ or} \\ c(w,y) &= \left[\frac{2}{\lambda} \sum_i \frac{\lambda}{2} \gamma_i + \sum_i \frac{\lambda}{2} \gamma_i w_i(\lambda) + \sum_j \frac{\lambda}{2} \beta_j w_j(\lambda) + \sum_i \sum_j \frac{\lambda}{2} \beta_{ij} w_i(\lambda) w_j(\lambda) \right]^{1/\lambda} y, \text{ or} \\ c(w,y) &= \left[\frac{2}{\lambda} \sum_i \sum_j \beta_{ij} + \sum_i \sum_j \beta_{ij} w_i(\lambda) + \sum_i \sum_j \beta_{ij} w_j(\lambda) + \sum_i \sum_j \frac{\lambda}{2} \beta_{ij} w_i(\lambda) w_j(\lambda) \right]^{1/\lambda} y, \text{ or} \\ c(w,y) &= \left\{ \frac{2}{\lambda} \sum_i \sum_j \beta_{ij} \left[1 + \frac{\lambda}{2} w_i(\lambda) + \frac{\lambda}{2} w_j(\lambda) + \frac{\lambda^2}{4} w_i(\lambda) w_j(\lambda) \right] \right\}^{1/\lambda} y, \text{ or} \\ c(w,y) &= \left[\frac{2}{\lambda} \sum_i \sum_j \beta_{ij} \left(\frac{\lambda}{2} w_i(\lambda) \right) = \left(\frac{\lambda}{2} w_j(\lambda) + 1 \right) \right]^{1/\lambda} y, \text{ or} \\ c(w,y) &= \left[\frac{2}{\lambda} \sum_i \sum_j \beta_{ij} (w_i w_j)^{\lambda/2} \right]^{1/\lambda} \cdot y. \end{aligned}$$

$$\text{Equation 2.71}$$

Derivation of Other Functional Forms

We can write Equation 2.67 as

$$\begin{aligned} c(w,y) &= [1 + \lambda G(w)]^{1/\lambda} y \quad \text{or} \\ \frac{\left(\left[\frac{c(w,y)}{y} \right]^\lambda - 1 \right)}{\lambda} &= G(w) \quad \text{or} \\ \frac{\left(\left[\frac{c(w,y)}{y} \right]^\lambda - 1 \right)}{\lambda} &= \gamma_o + \sum_{i=1}^n \gamma_i w_i(\lambda) + \frac{1}{2} \sum_i \sum_j \beta_{ij} w_i(\lambda) w_j(\lambda). \end{aligned} \quad \text{Equation 2.72}$$

Taking the limit as $\lambda \Rightarrow 0$ we get

$$\begin{aligned}\ln \frac{c(w,y)}{y} &= \gamma_o + \sum_{i=1}^n \gamma_i \ln w_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln w_i \ln w_j \\ \text{but } \frac{c(w,y)}{y} &= \frac{\text{total cost}}{\text{total output}} = \text{unit cost} \\ \ln u.c. &= \gamma_o + \sum_{i=1}^n \gamma_i \ln w_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln w_i \ln w_j\end{aligned}\quad \text{Equation 2.73}$$

which is a translog unit-cost-function.

$$\begin{aligned}\text{Now as } \lambda \rightarrow 0 \quad \sum \gamma_i &= 1 + \lambda \gamma_o \rightarrow 1 \\ \text{and } \sum \beta_{ij} &= \frac{\lambda}{2} \gamma_i \Rightarrow 0.\end{aligned}$$

Substituting into Equation 2.73, we get

$$\ln u.c. = \gamma_o + \sum_i \gamma_i \ln w_i \quad \text{Equation 2.74}$$

which is a unit cost CD function. Again, Equation 2.71 can be written as

$$\left(\frac{c(w,y)}{y} \right)^\lambda = \frac{2}{\lambda} \sum_i \sum_j \beta_{ij} (w_i w_j)^{\lambda/2}, \quad \text{Equation 2.75}$$

and, putting $\lambda=1$, we get

$$\frac{c(w,y)}{y} = u.c. = 2 \sum_i \sum_j \beta_{ij} (w_i w_j)^{1/2}, \quad \text{Equation 2.76}$$

which is a generalized Leontief unit cost function and, if $\beta_{ij} \neq 0$ for $i=j$, then Equation

2.76 becomes

$$u.c. = 2 \sum_i \beta_{ii} (w_i w_i)^{1/2} = 2 \sum_i \beta_{ii} w_i,$$

which is a Leontief unit cost function.

Similarly if we put $\lambda=2$ in Equation 2.75, we get

$$\begin{aligned} \left(\frac{c(w,y)}{y} \right)^2 &= \sum_i \sum_j \beta_{ij} w_i w_j \\ \text{or } \frac{c(w,y)}{y} &= u \cdot c = \sqrt{\sum_i \sum_j \beta_{ij} w_i w_j}, \end{aligned} \quad \text{Equation 2.77}$$

which is a quadratic unit cost function.

Again, if we put $\beta_{ij} = 0$, if $i \neq j$ in Equation 2.71 we get

$$\begin{aligned} u \cdot c &= \left[\frac{2}{\lambda} \sum_i \sum_j \beta_{ii} (w_i w_i)^{\lambda/2} \right]^{1/\lambda} \\ &= \left[\frac{2}{\lambda} \sum_i \beta_{ii} w_i^\lambda \right]^{1/\lambda}, \end{aligned} \quad \text{Equation 2.78}$$

which is constant-elasticity-of-substitution function (CES).

If we put $\lambda=2$ in Equation 2.78 we get

$$u \cdot c = \left[\sum_i \beta_{ii} w_i^2 \right]^{1/2} \quad \text{Equation 2.79}$$

In Equation 2.78 if we put

$$\beta_{ii} = \frac{\lambda}{2} \beta_{ii}^* \text{ and } \gamma_o = 0,$$

then we get

$$\sum_i \frac{2}{\lambda} \beta_{ii} = \sum_i \beta_{ii}^* = 1,$$

and therefore

$$u \cdot c = \left[\sum_i \beta_{ii}^* w_i^\lambda \right]^{1/\lambda}$$

Taking the limit as $\lambda \rightarrow 0$ we get

$$u \cdot c = w_i \beta_{ii}^*,$$

which is a CD function. In Equation 2.71 if we put $\lambda = -1$ we get

$$u \cdot c = \left[-2 \sum_i \sum_j \beta_{ij} (w_i w_j)^{-1/2} \right]^{-1} \text{ or}$$

$$\frac{1}{u \cdot c} = -2 \sum_i \sum_j \beta_{ij} (w_i w_j)^{-1/2} = -2 \sum_i \sum_j \frac{\beta_{ij}}{(w_i w_j)^{1/2}} ;$$

if

$$-2\beta_{ij} = \beta_{ij}^*,$$

then

$$\frac{1}{u \cdot c} (\text{inverse of unit cost}) = \sum_i \sum_j \frac{\beta_{ij}^*}{(w_i w_j)^{1/2}}$$

Equation 2.80

For $\lambda = -2$ we get

$$u \cdot c = \left[- \sum_i \sum_j \beta_{ij} (w_i w_j)^{-1} \right]^{-1/2}$$

$$\frac{1}{u \cdot c} (\text{inverse of unit cost}) = \left[- \sum_i \sum_j \beta_{ij} (w_i w_j)^{-1} \right]^{1/2}$$

$$= \left[\sum_i \sum_j \beta_{ij}^* / w_i w_j \right]^{1/2}, \text{ if } -\beta_{ij}^* = \beta_{ij}.$$

Equation 2.81

Technical Change and Returns to Scale

The generalized cost function can be modified to include scale and technical change effect. Thus the cost function of Equation 2.67 can be modified as

$$c(w, y, t) = [1 + \lambda G(w)]^{1/\lambda} y^{\alpha(\alpha, y, w)} e^{T(t, w, y)},$$

Equation 2.82

where

$$\alpha(\alpha, y, w) = \alpha + \frac{\theta}{2} \ln y + \sum_i \phi_i \ln w_i,$$

Equation 2.83

and α is a scale parameter which depends on output and inputs price w_i , and

$$T(t, w, y) = (\tau + \frac{\delta}{2} t + \sum_i \tau_i \ln w_i + \eta \ln y) t$$

Equation 2.84

where $T(t, w, y)$ is a technical change function dependent upon time, input variable prices and output. In terms of Hicks' (1963) induced invention hypothesis, the technical change is a response to market phenomena such as relative price changes, i.e., is a consequence of investment in the quality of labor and capital. The measurement of technical change is based on two approaches—(i) time trend is treated as continuous variable, and (ii) time is treated as discrete intervals. The generalized production function Equation 2.65 can be written with technical change as

$$y = [1 + \lambda F(x, t)]^{1/\lambda} \text{ or } \frac{y^\lambda - 1}{\lambda} = F(x, t).$$

Taking the limit as $\lambda \Rightarrow 0$ and differentiating w.r.t t gives

$$\frac{\partial \ln y}{\partial t} = \frac{\partial \ln F(x, t)}{\partial t} = T(x, t) \text{ rate of technical change.} \quad \text{Equation 2.85}$$

Technical change may be embodied or disembodied. Embodied technical change is the after-effect of new inputs with better qualities. Disembodied changes are productivity changes (improvements), while inputs qualities and quantities are kept constant.

Technical change can be progressive or regressive. A progressive technical change is one in which output expands and allows same input bundles to produce more than what was being produced. It is regressive if output shrinks for the same input bundles over the time period. That is, isoquants shift toward the origin or away from the origin in progressive and regressive technical change (Figure 2.7). In the third category, the passage of time generates an isoquant intersecting the previous one as shown in Figure 2.8. It is a sort of rotation of isoquants due to technical change which can be measured by

$$\frac{\partial \ln F(x,t)}{\partial t} = \frac{\partial \ln y}{\partial t} = T(x,t). \quad \text{Equation 2.86}$$

The rate of technical change is locally progressive if $T(x,t) \geq 0$, and is locally regressive if $T(x,t) < 0$.

Technical change can be neutral or non-neutral. As we know, in progressive/-regressive technical change, the isoquants shift towards or away from the origin. If this shift is such that the isoquants remain parallel to each other while shifting in or out, then the slopes of the isoquants remain constant or unchanged and hence the marginal rate of technical substitution is constant. This is equivalent to saying that MRTS is independent of time. Parallel shift demonstrates (see Figure 2.7) that technical change has not altered the relative marginal productivities of inputs because $\Delta \text{MRTS} = 0$, so that inputs either remain neutral during the technical change, or they are affected proportionately in equal amounts in such a way that $\Delta \text{MRTS} = 0$. Hence this type of technical change is either neutral progressive or neutral regressive depending on the sign of the inequality. This definition is according to Hicks (1963). Technical change will be non-neutral if the shift is non-parallel and $\Delta \text{MRTS} \neq 0$, i.e., not independent of time.

As neutral technical change (Hicks neutral) is characterized by a constant MRTS or constant slope of the isoquants, i.e., independent of time t , so we can define production as Hicks neutral if we can write $y = [1 + \lambda F(f(x), t)]^{1/\lambda}$, which is linearly homogenous in x , separable and homothetic.

From Equations 2.83 and 2.84, by partial differentiation w.r.t $\ln w_i$ we get

$$\frac{\partial \alpha(\alpha, y, w)}{\partial \ln w_i} = \sum \phi_i = 0$$

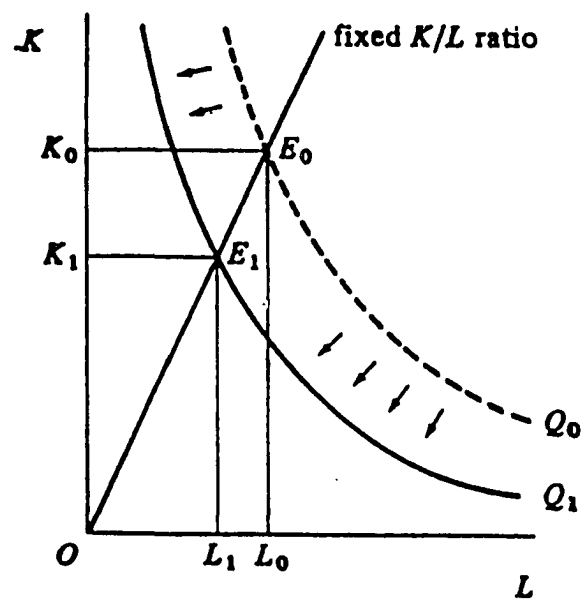


Figure 2.7 Homothetic shift of production function (Hebden, 1983, p. 147)

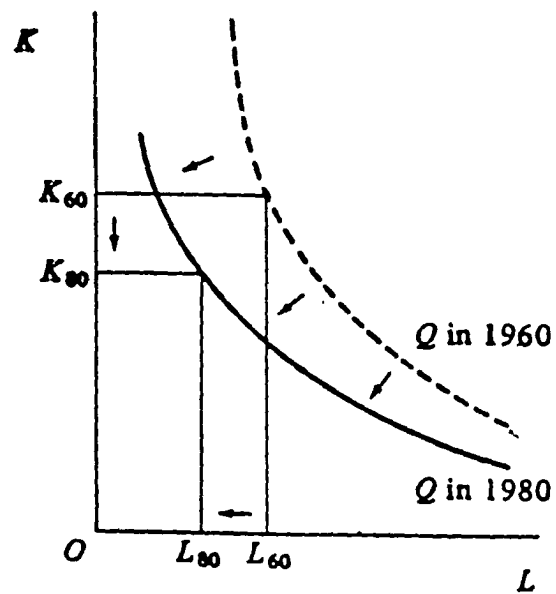


Figure 2.8 Non-homothetic shift of production function (Hebden, 1983, p. 146)

And

$$\frac{\partial T(t, w, y)}{\partial \ln w_i} = \sum \tau_i = 0 \Rightarrow \text{linear homogeneity in prices.} \quad \text{Equation 2.87}$$

If the cost function is to be consistent with homothetic technology, then it must factor into a function of output and one of prices. From Equation 2.83, if $\phi_i = 0$, then $c(w, y) = [1 + \lambda c(w)]^{1/\lambda} y^{\alpha(\alpha, y)}$ (omitting the technical change effect). It is clear from this equation that $c(w, y) = \hat{c}(w)f(y)$, which confirms homotheticity. Also, differentiation of Equation 2.83 w.r.t. $\ln y$ gives

$$\frac{\partial \alpha(\alpha, y, w)}{\partial \ln y} = \theta/2 = 0 \Rightarrow \text{the production function is homogeneous of degree } 1/\alpha \text{ in inputs.}$$

and

$$\frac{\partial \alpha(\alpha, y, w)}{\partial \alpha} = 1 \Rightarrow \text{constant returns to scale.}$$

In the case of Hicks neutral technical change, the MRTS is constant at the same point where the ratio of the factor costs is also constant, i.e., independent of time. Hence $\tau_i = 0$ in Equation 2.84 indicates cost-neutral technical change. Moreover, $\delta = 0$ and $\eta = 0$, give us the constant exponential form, Hicks neutral technical change, and shows zero interaction between time and scale of production. The only possibility of interaction is when the minimum point on the AC curve is either shifted to the right by the technical change for scale economies (Cowing, 1974) or increased scale operation may encourage technological innovation (Gold 1974). From Equations 2.71 and 2.82 we have

$$c(w, y) = \left[\frac{2}{\lambda} \sum_i \sum_j \beta_{ij} (w_i w_j)^{\lambda/2} \right]^{1/\lambda} y^{\alpha(\alpha, y, w)} e^{\pi(t, w, y)} \quad \text{Equation 2.88}$$

Taking the log of both sides and differentiating with respect to $\ln w_i$, the i th share cost

equation is obtained as follows:

$$S_i = \frac{\partial \ln c(w, y)}{\partial \ln w_i} = \frac{\sum_j \beta_{ij} (w_i w_j)^{\lambda/2}}{\sum_i \sum_j \beta_{ij} (w_i w_j)^{\lambda/2}} + \phi_i \ln y + \tau_i t, \quad \text{Equation 2.89}$$

where S_i is the cost share of input i in the total cost, while output and input prices are held constant.

Differentiation Equation 2.89 w.r.t. t and $\ln y$ we get

$$\frac{\partial S_i}{\partial t} = \tau_i \text{ and } \frac{\partial S_i}{\partial \ln y} = \phi_i. \quad \text{Equation 2.90}$$

If $\tau_i = 0$, then one has cost-neutral technical change, which acts exactly as equally-augmenting technology, that is, cost neutral technology, which means that cost minimizing input ratios are independent of the state of technology. On the other hand, if $\tau_i < 0$, then technological change decreases the i th share; that is, if technology is not cost neutral, the technical change is biased and causes a greater percentage adjustment in one input than in another, i.e., the cost-minimizing input ratios no longer are constant (independent of the state of technology).

Technical change is, therefore, unbiased (share neutral) if relative cost shares are constant, i.e.,

$$\begin{aligned} \frac{\partial (S_i/S_j)}{\partial t} &= 0 \quad \forall ij \\ \text{which is implied as} \\ \frac{\partial \ln \left(\frac{S_i}{S_j} \right)}{\partial t} &= 0 \quad \text{or} \quad \frac{\partial \ln S_i}{\partial t} = \frac{\partial \ln S_j}{\partial t} = \dots = \frac{\partial \ln S_n}{\partial t}. \end{aligned} \quad \text{Equation 2.91}$$

But under linear homogeneity of the cost function,

$$\frac{\partial}{\partial t} \sum_{i=1}^n \ln S_i = 0$$

which shows that cost shares are independent of state technology if technical change is unbiased (share neutral), i.e., share neutral technical change is equivalent to cost neutral technical change.

Technical changes is often said to be share "i" using or saving if

$$\frac{\partial \ln S_i}{\partial t} \begin{matrix} > \\ < \end{matrix} 0$$

Also, biased technical change can be defined in terms of the i th input demand factor $x_i(w, y, t)$, as

$$\begin{aligned} \frac{\partial x_i(w, y, t)}{\partial t} = 0 &\Rightarrow \text{neutral change} \\ > 0 &\Rightarrow \text{input using technical change} \\ < 0 &\Rightarrow \text{input saving technical change.} \end{aligned}$$

Similarly, non-homotheticity parameter ϕ_i indicates how economies of scale are distributed over various inputs. As

$$\frac{\partial S_i}{\partial \ln y} = \phi_i = 0, \forall i$$

implies homotheticity of the function and economies of scales are evenly distributed, and result in the same proportionate changes in all the input output coefficients. If $\phi_i > 0$ then an increase in output will result in large diseconomies in the i th input, i.e., the elasticity of cost with respect to output will increase

$$\left(\frac{\partial c / \partial y}{c / y} \right),$$

while if $\phi_i < 0$ an increase in output will result in large economies in the i th input, i.e.,

there is larger proportionate decline in per unit requirements of input, and elasticity will decrease.

$$\left(\frac{\partial c / \partial y}{c / y} \right)$$

In the above discussion the input prices w_i and time parameter t were held fixed.

Mathematically, we can write Equation 2.88 as

$$\begin{aligned} \frac{\partial S_i}{\partial \ln y} &= \phi_i = \frac{\partial}{\partial \ln y} \left(\frac{\partial \ln c}{\partial \ln w_i} \right) = \frac{\partial}{\partial \ln y} \left(\frac{w_i}{c} \frac{\partial c}{\partial w_i} \right) \\ &= \frac{\partial}{\partial \ln y} \left(\frac{w_i}{c} x_i \right), \left[\because \frac{\partial c}{\partial w_i} = x_i \right] \\ &= w_i \frac{\partial}{\partial \ln y} \left(\frac{x_i}{c} \right) \quad \text{when } w_i \text{ is fixed.} \\ &= w_i \left[c^{-1} \frac{\partial x_i}{\partial \ln y} + (-1) x_i c^{-2} \frac{\partial c}{\partial \ln y} \right] \\ &= \frac{w_i x_i}{c} \left[\frac{\partial x_i}{x_i \partial \ln y} - \frac{\partial c}{c \partial \ln y} \right] = S_i \left[\frac{\partial \ln x_i}{\partial \ln y} - \frac{\partial \ln c}{\partial \ln y} \right], \end{aligned} \quad \text{Equation 2.92}$$

where x_i =input i and c =total cost= $c(w,y)$.

For scale economies

$$\frac{\partial \ln c}{\partial \ln y} < 1, \text{ i.e., } = \frac{\partial c / \partial y}{c / y} < 1 \text{ or } \epsilon_{cy} < 1.$$

If

$$\begin{aligned} \phi_i &< 0, \\ \frac{\partial \ln x_i}{\partial \ln y} - \frac{\partial \ln c}{\partial \ln y} &< 0 \text{ or } \frac{\partial x_i / \partial y}{x_i / y} < \frac{\partial c / \partial y}{c / y} < 1 \text{ or} \\ \epsilon_{xy} &< \epsilon_{cy} < 1, \end{aligned} \quad \text{Equation 2.93}$$

which shows that the input-output coefficients will decrease as y is increasing.

As sufficient restrictions are imposed on the general form with scale and technical change, it is clear from the above discussion that elasticities of substitution, biases in technical change and returns to scales can be calculated. Thus the general form of the translog cost function can be obtained from Equation 2.82.

$$c(w,y) = [1 + \lambda G(w)]^{1/\lambda} y^{\alpha(\alpha,y,w)} e^{T(t,w,y)}$$

let

$$\hat{c} = c(w,y)/y^{\alpha(\alpha,y,w)} \cdot e^{T(t,w,y)}$$

then

Equation 2.94

$$\hat{c} = [1 + \lambda G(w)]^{1/\lambda} \text{ or } \frac{\hat{c}^\lambda - 1}{\lambda} = G(w)$$

Taking limit as

$$\lambda \rightarrow 0, (\hat{c}^\lambda - 1/\lambda) \rightarrow \ln \hat{c}$$

$$\therefore \ln \hat{c} = \lim_{\lambda \rightarrow 0} G(w),$$

But from Equation 2.94

$$\ln \hat{c} = \lim_{\lambda \rightarrow 0} G(w) - \ln y^{\alpha(\alpha,y,w)} - \ln e^{T(t,w,y)}$$

$$\text{or } \ln c(w,y) = \ln \hat{c} + \ln y^{\alpha(\alpha,y,w)} + \ln e^{T(t,w,y)}$$

$$\text{or } \ln c(w,y) = \lim_{\lambda \rightarrow 0} G(w) + \ln y^{\alpha(\alpha,y,w)} + \ln e^{T(t,w,y)}$$

or expanding $G(w)$ as second order Taylor approximation, we get

$$\begin{aligned} \ln c(w,y) &= \lim_{\lambda \rightarrow 0} \left[\gamma_o + \sum_i \gamma_i w_i(\lambda) + \frac{1}{2} \sum_i \sum_j \beta_{ij} w_i(\lambda) w_j(\lambda) \right] + \ln y^{\alpha(\alpha,y,w)} + \ln e^{T(t,w,y)} \\ &= \gamma_o + \sum_i \gamma_i \ln w_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln w_i \ln w_j + \ln y^{\alpha(\alpha,y,w)} + \ln e^{T(t,w,y)} \end{aligned}$$

because in the limit

$$w_i(\lambda) = \lim_{\lambda \rightarrow 0} \frac{w_i^{\lambda/2} - 1}{\lambda/2} = \ln w_i$$

or

$$\ln c = \gamma_o + \sum_i \gamma_i \ln w_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln w_i \ln w_j + \alpha(\alpha, y, w) \ln y + T(t, w, y) \quad \text{Equation 2.95}$$

which is a translog formulation. Similarly, CD, CES and other specific formulation can be obtained from general Equation 2.71, for different values of λ .

Comparative Statics of Generalized Function

The generalized cost function including technical change and scale effect is given in Equation 2.88, as

$$c(w, y) = \left[\frac{2}{\lambda} \sum_i \sum_j \beta_{ij} (w_i w_j)^{\lambda/2} \right]^{1/\lambda} y^{\alpha(\alpha, y, w)} e^{T(t, y, w)},$$

and is linearly homogeneous in input prices and symmetric ($\beta_{ij} = \beta_{ji}$). A sufficient condition for positivity of $c(w, y)$ is that for positive λ the parameter matrix of the cost function, i.e., $[\beta_{ij}]$ should be positive definite. Similarly, for negative λ , the $[\beta_{ij}]$ should be negative definite. A real symmetric matrix B is negative (positive) semi-definite if and only if

$$\dot{C}BC \leq 0 \text{ or } \dot{C}BC = 0, \text{ where } C = [c_{ij}] = \left[\frac{\partial^2 c(w, y)}{\partial w_i \partial w_j} \right]$$

If $\dot{C}BC < 0$ (strictly negative) or $\dot{C}BC > 0$ (strictly positive), then they are negative (positive) definite. Also a negative (positive) semidefinite matrix is negative (positive) definite only if its non-singular. If B is negative (positive) definite then B^{-1} (inverse) is also negative (positive) definite.

As we discussed earlier that for any concave (convex) function the Hessian matrix of the function should be negative (positive) definite. In our case $C=[c_{ij}]$ would be negative (positive) definite if C is concave (convex) in input prices. C will be concave (convex) in input prices if $\partial^2 c(w,y)/\partial w_i^2$ is either negative or positive.

From Equation 2.89, we see that

$$S_i = \frac{\partial \ln c(w,y)}{\partial \ln w_i} = \frac{\sum_j \beta_{ij}(w_i w_j)^{\lambda/2}}{\sum_i \sum_j \beta_{ij}(w_i w_j)^{\lambda/2}} + \phi_i \ln y + \tau_i t$$

and as long as $S_i \geq 0$, the function is non-decreasing in input prices. A sufficient condition for S_i to be non-negative is

$$\tau \geq 0$$

for all i,j .

The general cost function can be written as

$$c(w,y) = [1 + \lambda G(w)]^{1/\lambda} y^{\alpha(\alpha,y,w)} e^{T(t,w,y)} \text{ if } \hat{c} = \left[\frac{c(w,y)}{y^\alpha e^T} \right] \text{ then } \left(\frac{\hat{c}^\lambda - 1}{\lambda} \right) = G(w)$$

Taking limit as $\lambda \rightarrow 0$ gives

$$\ln \hat{c} = \lim_{\lambda \rightarrow 0} \ln G(w) \text{ or}$$

$$\begin{aligned} \ln c(w,y) &= \lim_{\lambda \rightarrow 0} \ln G(w) + \ln y^{\alpha(\alpha,y,w)} + \ln e^{T(t,w,y)} \\ &= \lim_{\lambda \rightarrow 0} \ln G(w) + \alpha(\alpha,y,w) \ln y + T(t,w,y) \end{aligned}$$

$$\ln c(w,y) = \lim_{\lambda \rightarrow 0} \ln G(w) + \alpha \ln y + \frac{\theta}{2} (\ln y)^2 + \sum \phi_i \ln w_i \ln y + \left(\tau + \frac{\delta}{2} t + \sum_i \tau_i \ln w_i + \eta \ln y \right) t$$

Equation 2.96

Now differentiating Equation 2.96 w.r.t $\ln y$ we get

$$\frac{\partial \ln c}{\partial \ln y} = \alpha + \theta \ln y + \sum \phi_i \ln w_i + \eta t > 0$$

Equation 2.97

for $\alpha, \theta, \eta > 0$ and $\phi_i > 0 \forall i$

and the second differential gives

$$\frac{\partial^2 \ln c}{\partial \ln y^2} = \theta \geq 0,$$

for $\theta \geq 0$

Usually the average cost curve is U-shaped, which means that it must attain a critical value y^* at which the first derivative is zero and second derivative is positive, with an increase in output (y) (See Figure 2.4). Mathematically,

$$\frac{\partial}{\partial y} \left(\frac{c(w, y^*)}{y^*} \right) = 0 \text{ and } \frac{\partial^2}{\partial y^2} \left(\frac{c(w, y^*)}{y^*} \right) \geq 0$$

Now from the above conditions we have

$$\begin{aligned} \frac{\partial}{\partial y} \left(\frac{c(w, y)}{y} \right) &= \frac{1}{y} \left[\frac{\partial c(w, y)}{\partial y} - \frac{c(w, y)}{y} \right] = \frac{c(w, y)}{y^2} \left(\frac{\partial c(w, y)/\partial y}{c(w, y)/y} - 1 \right) = 0 \Rightarrow \\ \frac{\partial c(w, y)/\partial y}{c(w, y)/y} &= 1 \end{aligned}$$

Equation 2.98

or L·H·S = $n(w, y)$ = cost flexibility or elasticity of cost w.r.t output or MC/AC.

We know that, for elasticity of size

$$\epsilon^* = \frac{1}{n(w, y)}, \quad \epsilon^*(w, y^*) = \frac{1}{n(w, y^*)}$$

From the second derivative condition evaluated at $y=y^*$,

$$\frac{\partial^2}{\partial y^2} \left(\frac{c(w,y)}{y} \right) = \frac{c(w,y^*)}{y^2} \cdot \frac{\partial n(w,y^*)}{\partial y} \geq 0 \rightarrow \frac{\partial n}{\partial y} \text{ is positive}$$

but $\frac{\partial n}{\partial y}$ is the reciprocal of ϵ^*

$$\therefore \frac{\partial \epsilon^*}{\partial y} < 0.$$

Hence the average cost curve will be U-shaped if $\epsilon^*(w,y) < 1$, i.e., the elasticity of size is decreasing for increasing y .

From the Equation 2.97 it is clear that the cost function is decreasing for increasing y , until $\theta=0$, at which point it attains minimum cost and the first derivative of the average cost function will vanish. At this point from Equation 2.98 we get the elasticity of size $\epsilon^*(w,y)$ which is reciprocal of elasticity of cost with respect to output $n(w,y)$ which is equal to 1. From Equation 2.97 we see that the second derivative is positive, which ensures that the average cost function reaches a minimum cost point as y is increasing, producing a U-shaped curve.

Different Elasticities of Generalized Cost Function

Elasticity of anything is defined as the ratio of the rate of change (partial derivative) of that thing to the unit change of what has caused this respective rate change.

Mathematically

$$\epsilon_i = \frac{\partial f(x)/\partial x_i}{f(x)/x_i}.$$

It can be written in the logarithmic form for any input x_i as:

$$\epsilon_i = \frac{\partial \ln f(x)}{\partial \ln x_i} \quad \text{Equation 2.99}$$

where $f(x)$ =output, x_i =input i , ϵ_i =elasticity of output with respect to input.

We may further define ϵ_{ii} =own elasticity and ϵ_{ij} =cross elasticity. For example the function $y=f(x)$ gives

$$\epsilon_{ii} = \frac{\partial y / \partial x_i}{y / x_i} = \frac{\partial \ln y_i}{\partial \ln x_i} \text{ and } \epsilon_{ij} = \frac{\partial y / \partial x_j}{y / x_j} = \frac{\partial \ln y_i}{\partial \ln x_j}. \quad \text{Equation 2.100}$$

For the cost function under discussion the own price elasticity is:

$$\epsilon_{ii} \text{ (own price elasticity)} = \frac{\partial \ln x_i}{\partial \ln w_i}. \quad \text{Equation 2.101}$$

But we know (according to Shephard's lemma) that

$$x_i = \frac{\partial c(w, y)}{\partial w_i}.$$

Hence

$$\begin{aligned} \epsilon_{ii} &= \frac{\partial \ln [\partial c(w, y) / \partial w_i]}{\partial \ln w_i} \\ \epsilon_{ii} &= \frac{w_i \partial^2 c(w, y) / \partial w_i^2}{\partial c(w, y) / \partial w_i} = w_i \frac{\Delta^2 c_i}{\Delta c_i}, \\ \Delta c_i &= \frac{\partial c}{\partial w_i} \text{ and } \Delta^2 c_i = \frac{\partial^2 c}{\partial w_i^2} \end{aligned} \quad \text{Equation 2.102}$$

For Equation 2.88 we have

$$\begin{aligned}
\hat{c} &= \left[\frac{2}{\lambda} \sum_i \sum_j \beta_{ij} (w_i w_j)^{\lambda/2} \right]^{1/\lambda} & \hat{c} &= \frac{c}{y} \frac{\alpha(\alpha, y, w) T(w, y, t)}{e} \\
\epsilon_{ii} &= w_i \frac{\partial^2 c(w, y) / \partial w_i^2}{\partial c(w, y) / \partial w_i} = w_i \Delta^2 c_i / \Delta c_i & \frac{\partial \hat{c}}{\partial w_i} &= \frac{\partial}{\partial w_i} \left(\frac{c}{y^{\alpha(w, y, \alpha)} e^{T(w, y, H)}} \right) \\
\frac{\partial \hat{c}}{\partial w_i} &= \frac{2w_i}{\lambda \hat{c}^{\lambda-1}} \sum_i \beta_{ii} (w_i w_j)^{\lambda/2-1} & \frac{\partial^2 \hat{c}}{\partial w_i^2} &= \frac{\partial^2}{\partial w_i^2} \left(\frac{c}{y^{\alpha} e^T} \right) \\
\frac{\partial^2 \hat{c}}{\partial w_i^2} &= \frac{\left(\frac{\lambda}{2} - 1 \right) w_j \sum \beta_{ii} (w_i w_j)^{\lambda/2-2} - 2 \frac{(\lambda-1) w_i}{\lambda \hat{c}^{\lambda}} \left(\sum_i \beta_{ii} (w_i w_j)^{\lambda/2-1} \right)^2}{\sum_i \beta_{ij} (w_i w_j)^{\lambda/2-1}}.
\end{aligned}$$

Equation 2.103

Rearranging the terms and putting

$$S_i = \frac{\sum_i \beta_{ij} (w_i w_j)^{\lambda/2}}{\sum_i \sum_i \beta_{ij} (w_i w_j)^{\lambda/2}} + F_i(y, t)$$

Equation 2.104

where

$$F_i(y, t) = \phi_i \ln y + \tau_i t$$

we get

$$\epsilon_{ii} = (1-\lambda) S_i + \beta_{ii} \frac{(w_i / \hat{c})^{\lambda}}{S_i} + \lambda F_i(y, t) + \frac{\lambda}{S_i} \{ (S_i - F_i(y, t)) \} F_i(y, t) + \frac{\lambda}{2} \left\{ 1 - \frac{F_i(y, t)}{S_i} \right\} - 1$$

Equation 2.105

If progress is neutral and the cost function homothetic, then $F_i(y, t) \Rightarrow 0 \therefore \phi \Rightarrow 0$

and $\tau \Rightarrow 0$, hence Equation 2.105 reduces to the form

$$\epsilon_{ii} = (1-\lambda) S_i + \beta_{ii} \frac{(w_i / \hat{c})^{\lambda}}{S_i} + \frac{\lambda}{2} - 1$$

Equation 2.106

The cross price elasticity is

$$\epsilon_{ij} = \frac{\partial \ln x_i}{\partial \ln w_j} = \frac{\partial \ln \left(\frac{\partial c(w, y)}{\partial w_i} \right)}{\partial \ln w_j}. \quad \text{Equation 2.107}$$

Further, for the cost function under consideration, from 2.103

$$\epsilon_{ij} = (1-\lambda)S_j + \beta_{ij} \frac{(w_i w_j)^{\lambda/2}}{S_i \hat{c}^\lambda} + \lambda F_j(y, t) + \frac{\lambda}{S_i} (S_j - F_j(y, t) F_i(y, t)) \quad \text{Equation 2.108}$$

For homothetic and neutral change we get

$$\epsilon_{ij} = (1-\lambda)S_j + \beta_{ij} \frac{(w_i w_j)^{\lambda/2}}{S_i \hat{c}^\lambda}. \quad \text{Equation 2.109}$$

As mentioned before, Allen partial elasticities can be found by dividing the own and cross elasticities by the cost share.

Thus $\sigma_{ii} = \epsilon_{ii}/S_i$ and $\sigma_{ij} = \epsilon_{ij}/S_j$ where

$$S_i = \frac{\partial \ln c}{\partial \ln w_i}, \quad S_j = \frac{\partial \ln c}{\partial \ln w_j}. \quad \text{Equation 2.110}$$

We can find elasticities for different functional forms by incorporating the corresponding restrictions in the above equations.

1. Translog Function: Put $\lambda=0$ in Equations 2.106 and 2.109 to obtain

$$\begin{aligned} \epsilon_{ii} &= S_i + \beta_{ii}/S_i - 1 \text{ (own)} \\ \epsilon_{ij} &= S_j + \beta_{ij}/S_i \text{ (cross)} \end{aligned} \quad \text{Equation 2.111}$$

2. Cobb-Douglas: Putting $\beta_{ij} = 0$ in Equation 2.111 gives

$$\begin{aligned} \epsilon_{ii} &= S_i - 1 \\ \epsilon_{ij} &= S_j \end{aligned} \quad \text{Equation 2.112}$$

3. Constant Elasticity of Substitution: $\beta_{ij} = 0$ at $i \neq j$ in the generalized function gives the own and cross price elasticities of the constant elasticity of substitution function as

and

$$\begin{aligned}\epsilon_{ii} &= (1-\lambda)S_i + \lambda/2 + \lambda/2 - 1 = (S_i-1)(1-\lambda) \\ \epsilon_{ij} &= (1-\lambda)S_j\end{aligned}\quad \text{Equation 2.113}$$

$$\left[\therefore \frac{\beta_{ii}(w_i/\partial)^{\lambda}}{S_i} \rightarrow \frac{\lambda}{2} \text{ as } L.H.S = \frac{\beta_{ii}w_i^{\lambda/2}}{\frac{2}{\lambda} \sum_j \beta_{ij}w_j^{\lambda/2}} \Rightarrow \frac{\lambda}{2} \text{ at } i=j \right]$$

4. Leontief Cost Function: Putting $\lambda = 1$ in Equation 2.113, we get $\epsilon_{ii} = 0$, $\epsilon_{ij} = 0$, because, for the Leontief cost function, $c(w,y) = y \min w_i/\beta_i$. Differentiating with respect to w_i (input prices),

$$\frac{\partial c(w,y)}{\partial w_i} = y/\beta_i = x_i \text{ (cost minimizing derived demand)}$$

Now

$$\epsilon_{ii} = \frac{\partial \ln x_i}{\partial \ln w_i} = w_i \frac{\partial^2 c(w,y)/\partial w_i^2}{\partial c(w,y)/\partial w_i}$$

As

$$x_i = \frac{\partial c(w,y)}{\partial w_i},$$

Therefore

$$\frac{\partial x_i}{\partial w_i} = \frac{\partial^2 c(w,y)}{\partial w_i^2} = \frac{\partial (y/\beta_i)}{\partial w_i} = 0$$

Equation 2.114

Hence $\epsilon_{ii} = 0$; similarly $\epsilon_{ij} = 0$.

5. Generalized Leontief Function: Putting $\lambda=1$ in the generalized function Equations 2.106 and 2.109, we get

$$\epsilon_{ii} = \frac{1}{2} \left[\frac{\beta_{ii}(w_i)^{1/2}}{\sum_j \beta_{ij} w_j^{1/2}} - 1 \right] \text{ and } \epsilon_{ij} = \frac{1}{2} \frac{\beta_{ij}(w_j)^{1/2}}{\sum_j \beta_{ij}(w_j)^{1/2}} \quad \text{Equation 2.115}$$

6. Quadratic Function: Putting $\lambda=2$ in the generalized function Equations 2.106 and 2.109, we get

$$\epsilon_{ii} = \frac{\beta_{ii}(w_i)^{1/2}}{\sum_j \beta_{ij} w_j^{1/2}} - S_j \text{ and } \epsilon_{ij} = \frac{\beta_{ij}(w_j)^{1/2}}{\sum_j \beta_{ij}(w_j)^{1/2}} - S_j.$$

From the above special forms we can find the Allen elasticities by dividing the respective expression by S_i and S_j , the i th and j th cost shares.

Approximation Properties of the Generalized Function

The given cost function c^* can be approximated to c by the generalized function because of its free parameters. According to Diewert (1974b), an arbitrary twice-derivable cost function c^* at given values of w^* , y^* and t^* can be approximated as $c^* = c^*(w^*, y^*, t^*) = c(w^*, y^*, t^*)$. For given value of λ , symmetry of β and linear homogeneity of prices, the generalized function has $(n+2)(n+3)/2$ free parameters, which can be chosen to satisfy the following equations:

$$(a) \ c_i^*(w^*, y^*, t^*) = c_i(w^*, y^*, t^*) \text{ or}$$

$$c_i^* = c_i = \frac{\partial c}{\partial w_i} = \left[\sum_j \frac{\beta_{ij}(w_i w_j)^{\lambda/2}}{\sum_i \sum_j \beta_{ij}(w_i w_j)^{\lambda/2}} + \phi_i \ln y + \tau_i t \right] \frac{c}{w_i} \quad \text{Equation 2.116}$$

$i=1, \dots, n$

$$\begin{aligned}
(b) \quad c_{ij}^*(w^*, y^*, t^*) &= c_{ij}(w^*, y^*, t^*) \\
c_{ij}^* = c_{ij} &= \frac{\partial^2 c}{\partial w_i \partial w_j} = \epsilon_{ij} \frac{c_i}{w_j} = \frac{c_i}{w_j} \left((1-\lambda) S_j + \beta_{ij} \frac{(w_i w_j)^{\lambda/2}}{S_i \tilde{c}^\lambda} \right) \\
\text{or } c_{ij}^* &= (\epsilon_{ij}/w_j)(\partial c/\partial w_i) \text{ for } 1 \leq i \leq j \leq n
\end{aligned}
\tag{Equation 2.117}$$

$$\begin{aligned}
(c) \quad c_{iy}^*(w^*, y^*, t^*) &= c_{iy}(w^*, y^*, t^*) \\
\text{or } c_{iy}^* = c_{iy} &= \frac{\partial^2 c}{\partial w_i \partial y} = \frac{\partial c/\partial w_i}{y} \left[\beta + \theta \ln y + \sum_i \phi_i \ln w_i + \eta t + \phi_i c/w_i \left(\frac{\partial c}{\partial w_i} \right) \right] \\
\text{for } i &= 1, \dots, n
\end{aligned}
\tag{Equation 2.118}$$

$$\begin{aligned}
(d) \quad c_{it}^* = c_{it} &= \frac{\partial^2 c}{\partial w_i \partial t} = \frac{\partial c}{\partial w_i} \left[\tau + \delta t + \sum_i \tau_i \ln w_i + \eta/n y + \tau_i c/w_i \left(\frac{\partial c}{\partial w_i} \right) \right] \\
&\text{for } i = 1, \dots, n
\end{aligned}
\tag{Equation 2.119}$$

$$(e) \quad c_{yy}^* = c_{yy} = \frac{\partial^2 c}{\partial y^2} = c_y^2/c + \theta c/y^2 - c_y/c
\tag{Equation 2.120}$$

$$(f) \quad c_{tt}^* = c_{tt} = \frac{\partial^2 c}{\partial t^2} = c_t^2/c + \delta c
\tag{Equation 2.121}$$

$$(g) \quad c_{yt}^* = c_{yt} = \frac{\partial^2 c}{\partial y \partial t} = \frac{c_y c_t}{c} + \eta c/y
\tag{Equation 2.122}$$

In the above equations c_i^* is the derivative of the given cost function to be approximated, and c is the generalized cost function; also ϵ_{ij} is cross price elasticity. If the parameters of the generalized function are chosen such that the above equations are satisfied, then the second order differential approximation requirements are also satisfied.

Total Factor Productivity, Rate of Technical Change and Returns to Scale

In the case of the production function, the total factor productivity is defined as the average product of all inputs. If y is output and x is input vector, then total factor productivity (TFP) = y/x . Differentiating both sides of the equation w.r.t to time logarithmically we get

$$d\ln(TFP)/dt = d\ln y/dt - d\ln x/dt \quad \text{Equation 2.123}$$

Now the time rate of change of aggregate input ($d\ln x/dt$) is equal to the sum of time rates of changes of individual inputs, weighted by average cost shares ($w_j x_j/c$), and is equal to

$$\sum_j (w_j x_j/c) \text{ or } d\ln x/dt = \sum_j (w_j x_j/c) d\ln x_j/dt. \quad \text{Equation 2.124}$$

From Equation 2.123

$$d\ln(TFP)/dt = d\ln y/dt - \sum_j \left(\frac{w_j x_j}{c} \right) d\ln x_j/dt \quad \text{Equation 2.125}$$

On the other hand, technical change (TC) is defined as the shift of the production function's isoquant over the passage of time or, mathematically, it gives a relationship between output, input and time, i.e., $y=f(x,t)$ and TC measures how y changes with increase in t , keeping x constant. TC is given by differentiating the above equation logarithmically w.r.t. 't'

$$\therefore T(x,t) = \partial \ln f(x,t) / \partial t$$

The concept input-augmenting or factor-augmenting technical change express how an input makes a difference in actual production, rather than just shifting the production isoquant with the passage of time. These concepts measure the improvement in input

efficiency, and may not be treated as embodied TC (discussed earlier), because the stable relationship of output, input and time still exists, though the effectiveness of inputs varies over time (e.g., learning by doing).

Let the effective input vector be \bar{x} , a function of (x, t) ; then

$$y = f(\bar{x}(x, t), t). \quad \text{Equation 2.126}$$

Differentiating Equation 2.126 logarithmically with respect to t , we get

$$\partial \ln y / \partial t = \sum \frac{\partial \ln f}{\partial \ln \bar{x}_i} \cdot \frac{\partial \ln \bar{x}_i}{\partial t} + \frac{\partial \ln f}{\partial t} \quad \text{Equation 2.127}$$

Let $\tilde{\epsilon}_i$ = elasticity of output with respect to \bar{x} ; then

$$\tilde{\epsilon}_i = \frac{\partial \ln f}{\partial \ln \bar{x}_i}$$

$$\tilde{\epsilon} = \text{elasticity of scale with } \bar{x} = |\partial \ln(\lambda \bar{x}) / \partial \ln \lambda|_{\lambda=1} \text{ and } \theta_i = \frac{\tilde{\epsilon}_i}{\tilde{\epsilon}}.$$

We can write the above Equation 2.127 as

$$\partial \ln y / \partial t = \tilde{\epsilon} \sum \theta_i \frac{\partial \ln \bar{x}}{\partial t} + \frac{\partial \ln f}{\partial t} \text{ or}$$

$$\frac{\partial \ln f(x, t)}{\partial t} = \frac{\partial \ln y}{\partial t} - \tilde{\epsilon} \sum \theta_i \frac{\partial \ln \bar{x}}{\partial t} = T(x, t) \quad \text{Equation 2.128}$$

Equation 2.128 is made up of two parts, i.e., (i) the first part is a pure functional shift and can't be attributed to any particular input; (ii) the second part is composed of scale expansion effects given by the elasticity of scale times a weighted average of time rates of change of the various effective inputs.

The concept of Harrod neutrality is generalized by input-augmenting technical change. As we have seen, for the two-factor input, Hicks neutrality postulates that the marginal rate of technical substitution is independent of time t , while the Harrod

neutrality postulates that only labor effectiveness changes over time, implying that all the augmentation factors except labor are equal to one.

For the cost function with time variable 't', i.e., $c(w,y,t)$, differentiable in w , the behavior of $c(w,y,t)$ is non-decreasing in 't', if TC is progressive, and is non-increasing in 't' if TC is regressive.

A unique relationship exists between ΔTC , ϵ (elasticity of size) and the derivative of the cost function w.r.t time, is known as rate of cost diminution 'd', and is explored as follows.

Let

$$d(w,y,t) = \frac{\partial \ln c(w,y,t)}{\partial t} \quad \text{Equation 2.129}$$

the rate of cost diminution.

Now consider the Lagrangian of the cost function with fixed output:

$$L(c,t) = w \cdot x + q(y - f(x,t)), \quad q = \text{Lagrange multiplies}$$

Now by the envelope theorem and Kuhn-Tucker condition

$$\frac{\partial L}{\partial t} = \partial c(w,y,t)/\partial t = - q \partial f(x,t)/\partial t \quad \text{Equation 2.130}$$

The optimal value of the multiplier q , in cost minimization, equals the marginal cost;

hence

$$q = \partial c/\partial y. \quad \text{Equation 2.131}$$

Putting this value into Equation 2.130, we get

$$\partial c(w,y,t)/\partial t = - \partial c/\partial y \cdot \frac{\partial f(x,t)}{\partial t}.$$

Comparing Equation 2.133 with Equation 2.125 we see that

$$TFP = T(x,t), \text{ if } \epsilon^* = \text{constant} \quad \text{Equation 2.134}$$

This shows that rate of change of TFP is equal to TC (technical change), when elasticity of size is one or production is characterized by constant returns to scale.

In the case of the generalized function

$$d(w,y,t) = \partial \ln c / \partial t = (\tau + \delta t + \sum \tau_i \ln w_i + \eta \ln y) \quad \text{Equation 2.135}$$

and

$$\epsilon^*(w,y,t) = \frac{\partial \ln y}{\partial \ln c} = \left[\alpha + \theta \ln y + \sum_i \phi_i \ln w_i + \eta t \right]^{-1}. \quad \text{Equation 2.136}$$

Then

$$(TFP) = d(w,y,t) \epsilon^*(w,y,t), \quad \text{Equation 2.137}$$

which shows that rate of change of total factor productivity can be calculated parametrically.

Sensitivity Analysis

In the following paragraphs, we analyze the response to changes in output, time, price, scale effects, cost diminution and total factor productivity.

From Equation 2.90 we know that the time rate change of the i th cost share is equal to the non-neutrality parameter τ_i and the logarithmic output change rate of the i th cost share is equal to the non-homotheticity parameter ϕ_i . Mathematically

$$\frac{\partial S_i}{\partial t} = \begin{array}{ll} < 0 \rightarrow \text{input saving (progressive)} & TC \\ \tau_i = 0 \rightarrow \text{input neutral (neutral)} & TC \\ > 0 \rightarrow \text{input using (regressive)} & TC \end{array}$$

$$\text{Also } \frac{\partial S_i}{\partial \ln y} = \phi_i = \begin{cases} < 0 \rightarrow \text{economies of scale} \\ = 0 \rightarrow \text{indifferent (homothetic)} \\ > 0 \rightarrow \text{diseconomies of scale} \end{cases}$$

From Equation 2.89 we have by partial differentiation with respect to $\ln w_i$

$$\frac{\partial S_i}{\partial \ln w_i} = (\sigma_{ii} - 1)S_i^2 + S_i \quad \text{Equation 2.138}$$

and with respect to $\ln w_j$

$$\frac{\partial S_i}{\partial \ln w_j} = (\sigma_{ij} - 1)S_i S_j \quad \text{Equation 2.139}$$

where σ_{ii} , σ_{ij} are defined as before.

Now let us see how 'd', ϵ^* and ΔTFP react to changes in output 'y' and the time variable, 't'.

Differentiating Equations 2.135, 2.136, and 2.137 with respect to $\ln y$ and t , we get

$$\frac{\partial d(w,y,t)}{\partial \ln y} = -\eta \quad \text{Equation 2.140}$$

$$\frac{\partial \epsilon^*(w,y,t)}{\partial \ln y} = -\epsilon^{*2}\theta \quad \text{Equation 2.141}$$

and

$$\begin{aligned} \frac{\partial (TFP)}{\partial \ln y} &= -[d(w,y,t)(-\epsilon^{*2}\theta) + \epsilon^*(w,y,t)(-\eta)] \\ &= \epsilon^*(w,y,t)(d(w,y,t)\epsilon^*(w,y,t)\theta + \eta) \\ &= -\epsilon^*(w,y,t)(TFP(\theta) + \eta). \end{aligned} \quad \text{Equation 2.142}$$

Also

$$\frac{\partial d(w,y,t)}{\partial t} = -\delta \quad \text{Equation 2.143}$$

$$\frac{\partial \epsilon^*(w, y, t)}{\partial t} = \epsilon^{*2} \eta \quad \text{Equation 2.144}$$

$$\begin{aligned} - \frac{\partial(TFP)}{\partial t} &= - [d(w, y, t)(-\epsilon^{*2} \eta) + \epsilon^*(-\delta)] \\ &= +\epsilon^*(w, y, t)[d(w, y, t)\epsilon^*(w, y, t)\eta + \delta] \\ &= -\epsilon^*(w, y, t)[(TFP)(\eta) + \delta] \end{aligned} \quad \text{Equation 2.145}$$

$$\frac{\partial d}{\partial \ln w_i} = -\tau_i \quad \text{Equation 2.146}$$

$$\frac{\partial \epsilon^*}{\partial \ln w_i} = -\phi_i \epsilon^{*2} \quad \text{Equation 2.147}$$

$$\frac{\partial(TFP)}{\partial \ln w_i} = -(\tau_i + \phi_i(TFP))\epsilon^*. \quad \text{Equation 2.148}$$

If $\eta < 0$, then from Equation 2.140 we see that the 'd' will increase with increase in y (output). This is possible for large-scale plant operation.

If $\theta > 0$, then Equation 2.141 gives us that ϵ^* will decline as y is increased, that is, returns to scale will decrease. As we discussed earlier this decrease will continue until the minimum average cost point is attained, at which $\epsilon^* = 1$ and returns to scale will become constant.

From Equation 2.142 it is not clear what will happen to TFP when $\eta < 0$ and TFP > 0 as output will increase.

In Equation 2.143, if $\delta < 0$, then 'd' will increase with time, while in Equation 2.144 if $\eta < 0$, then ϵ^* (returns to scale) will be increased by TC. Also, the rate of TFP will increase with time if $\delta < 0$, $\eta < 0$ and TFP > 0 [From Equation 2.145].

From the last three equations we see that if $\tau_i < 0$ (input i saving TC), an increase in input price w_i , will increase 'd', and for $\phi_i < 0$, the economies of scale related to input i will result in increasing returns to scale with increased price of input i , while total factor productivity will also increase with increasing w_i , provided ϕ_i and τ_i are negative and $TFP > 0$.

Other productivity indexes are average product as defined in Equation 2.5, i.e., $AP_i = y/x_i$. Since $x_i = \partial c / \partial w_i$, the average product productivity index will be affected by the relative price changes, irrespective of the fact that scale economies are homothetic and technical change may be neutral.

For the non-neutral and non-homothetic case the partial productivity indexes are given as

$$\begin{aligned} (i) \quad & \frac{\partial \ln(y/x_i)}{\partial \ln y} = [\text{for output variation and input prices } w_i \text{ are fixed}] \\ (ii) \quad & \frac{\partial \ln(y/x_i)}{\partial t} = (\text{time rate, input prices } w_i \text{ and output fixed}) \\ (iii) \quad & \frac{\partial \ln(y/x_i)}{\partial \ln w_i} = (\text{input price changes}) \end{aligned}$$

The expression (i) can be evaluated as

$$\frac{\partial \ln(y/x_i)}{\partial \ln y} = \frac{\partial \ln y}{\partial \ln y} - \frac{\partial \ln x_i}{\partial \ln y} = 1 - \frac{\partial \ln x_i}{\partial \ln c} \cdot \frac{\partial \ln c}{\partial \ln y} = 1 - \epsilon_y \left(\frac{\partial \ln x_i}{\partial \ln c} \right)$$

Now

$$S_i = \frac{w_i x_i}{c} \therefore \ln S_i = \ln w_i + \ln x_i - \ln c$$

Differentiating with respect to $\ln c$,

$$\frac{\partial \ln S_i}{\partial \ln c} = 0 + \frac{\partial \ln x_i}{\partial \ln c} - 1 \text{ or } \frac{\partial \ln x_i}{\partial \ln c} = 1 + \frac{\partial \ln S_i}{\partial \ln c} \therefore \frac{\partial \ln(y/x_i)}{\partial \ln y} = 1 - \epsilon_{cy} - \epsilon_{cy} \cdot \frac{\partial \ln S_i}{\partial \ln c} \quad \text{Equation 2.149}$$

Now

$$\frac{\partial \ln S_i}{\partial \ln c} = \frac{\partial S_i / \partial c}{S_i / c} = \frac{\frac{\partial S_i}{\partial \ln y} \cdot \frac{\partial \ln y}{\partial c}}{S_i / c} = \frac{\phi_i}{S_i} \frac{\partial \ln y}{\partial c / c} = \frac{\phi_i}{S_i} \frac{\partial \ln y}{\partial \ln c} = \frac{\phi_i}{S_i} \frac{1}{\epsilon_{cy}}$$

Putting this in Equation 2.149 we get

$$\frac{\partial \ln(y/x_i)}{\partial \ln y} = 1 - \epsilon_{cy} - \phi_i / S_i. \quad \text{Equation 2.150}$$

Similarly (ii) is equal to

$$\frac{\partial \ln(y/x_i)}{\partial t} = d - \tau_i / S_i, \quad \text{Equation 2.151}$$

and (iii) is

$$\frac{\partial \ln(y/x_i)}{\partial \ln w_i} = - \epsilon_{ii}, \quad \text{Equation 2.152}$$

and also

$$\frac{\partial \ln(y/x_i)}{\partial \ln w_j} = - \epsilon_{ij}. \quad \text{Equation 2.153}$$

In the above equations, the scale effect of average product will increase with increase in y (output) if $\phi_i < 0 \rightarrow$ economies of scale. On the other hand, if $\phi > 0$ then average product will decrease \rightarrow diseconomies of scale. However $\phi_i=0$, will be a homothetic case. The average product increase or decrease will also depend on the input shares (S_i). Similarly the increase of the average product is dependent upon the time effect. As also seen above in the case of TFP, if $\tau_i < 0$, then progressive technical change or input i

saving TC is experienced, while, if $\tau_i > 0$ then the average product will decrease and hence technical change will be input i using or regressive. The effect of input price w_i on the average product is given by the negative of the price elasticities.

CHAPTER III. DESIGN OF EMPIRICAL MODEL TESTING

This chapter deals with the issues and problems encountered in empirical implementation of the generalized model discussed in the last chapter. In particular, the system of estimating equations, their stochastic representation, econometric issues, the testing procedure and measures of fit will be discussed in this chapter.

System of Estimating Equations

There are two problems with the direct estimation of the cost function alone. The first one is that of multi-collinearity and the other is that of optimizing behavior. Multicollinearity changes to perfect collinearity when the data matrix has less than full rank, and becomes singular and non-invertible due to linear dependency of columns. The problem of multicollinearity occurs when the data matrix is not singular but is nearly so. The symptoms are very low 't' ratios accompanying high 'F' ratios and high values of R^2 . In such situations the remedy is to augment the sample data by data exhibiting significant differences from the data already available, or to delete unrepresentative data, such as war years. Another way is to provide information directly on some of the parameters to be estimated.

The second approach for treating multicollinearity is to scale down the model. The difficulty of this approach is knowing which specific variable might be removed, and judicious choices must be made in respecifying the model. High simple or partial correlation between two variables will indicate multicollinearity but low values of such correlations do not negate it (Chipman 1964).

The third approach is to leave the model as it is, because scaling down or upgrading the model may induce the specification error.

The consideration of multicollinearity is also dependent upon the purpose of the study. For example, it is not a big problem in the case of forecasting because the same relationship among explanatory variables usually exists in the forecast period. On the other hand, for structural analysis it is a problem and must be taken care of.

The single equation of the cost function can not fully incorporate the notion of optimizing behavior of all relevant economic agents; hence factor demand equations or factor share equations should be included in the estimation, to provide additional information.

The generalized cost function with scale and technical change effects is as given in the Equation 2.82:

$$c(w, y, t) = [1 + \lambda G(w, y)]^{1/\lambda} y^{\alpha(\alpha, y, w)} e^{\pi(t, w, y)}, \quad \text{Equation 3.1}$$

where

$$G(w) = \gamma_o + \sum_i^n \gamma_i w_i(\lambda) + \frac{1}{2} \sum_i^n \sum_j^n \beta_{ij} w_i(\lambda) w_j(\lambda), \quad i=0-n, \quad j=1-n, \quad \text{Equation 3.2}$$

$$w_i(\lambda) = \begin{cases} (w_i^{\lambda/2} - 1)/\lambda/2 & \text{when } \lambda \neq 0 \\ \ln w_i & \text{when } \lambda = 0 \end{cases}, \quad w_i = \text{input price}, \quad \text{Equation 3.3}$$

$$\alpha(\alpha, y, w) = \alpha + \frac{\theta}{2} \ln y + \sum_i \phi_i \ln w_i, \quad i = 0 - n, \quad \text{Equation 3.4}$$

α =scale parameter, θ =neutrality parameter, ϕ_i =homotheticity parameter,

and

$$T(t, w, y) = (\tau + \sum_i \tau_i \ln w_i) t, \quad i = 0 - n. \quad \text{Equation 3.5}$$

The difference between this equation and Equation 2.84 of the second chapter is that η and δ , in Equation 3.5, are taken as zero, expressing the absence of interaction between scale and time, and incorporating the constant exponential form of Hicks neutral technical change.

The share equation system can be obtained by partial differentiation of Equation 3.1 with respect to input prices w_i . From Equation 2.89 we have

$$S_i = \frac{\partial \ln c(w, y, t)}{\partial \ln w_i} = \frac{\sum_j \beta_{ij} (w_i w_j)^{\lambda/2}}{\sum_i \sum_j \beta_{ij} (w_i w_j)^{\lambda/2}} + \phi_i \ln y + \tau_i t \quad \text{Equation 3.6}$$

The restrictions of symmetry, linear homogeneity, homotheticity and neutrality are imposed on Equation 3.6 for the shares S_i . For other types of functions, the share equations can be obtained with the corresponding value of the transformation parameter λ . For example, in the case of the translog function, the value of the Equation 3.6 as $\lambda \rightarrow 0$ in the limiting case is given by

$$S_i = \gamma_i + \sum_j \beta_{ij} \ln w_j + \phi_i \ln y + \tau_i t \quad \text{Equation 3.7}$$

On the basis of equations 3.1-3.6, the test for linear homogeneity and symmetry in input prices, homotheticity of function and neutrality of technical change can be done parametrically and separately.

If there are no restrictions on the systems of equation, then we have from Equations 3.1-3.5, the number $(n+1)n + n + 3 + n + 2 = n^2 + 3n + 5$ of free parameters. We

Table 3.1. Number of restriction and free parameter in the model

Number of Parameter	n Factor Model	4 Factor Model
Total Number	$(n+1)n+n+3+n+2$	33
1. Impose symmetry and homogeneity.	$n/2(n+1)+n+1+n+2$ [$\beta_{ij}=\beta_{ji}, \Sigma\gamma_i=1+\lambda\gamma_o$]	20
2. Also homotheticity, and 1.	$n/2(n+1)+1+n+2$ [Also $\Sigma\phi_i=0$]	17
3. Also neutrality, and 1.	$n/2(n+1)+n+1+2$ [$\Sigma\tau_i=0$]	17
4. Homotheticity, neutrality and 1.	$n/2(n+1)+1+2$	13
5. Homogeneity in output, and 3.	$n/2(n+1)+1+2$	13
6. Constant returns, and 3.	$n/2(n+1)+2$	12

can see below that the number of free parameters is reduced gradually as the number of restrictions is increased slowly in the general factor ($n=4$ in our case).

Stochastic Framework of the Model

The proper form of the model estimation is to set up the systems of equations into a stochastic framework. The deviation of the real optimal values from the observed value is considered by the researcher to be due to many types of error. The error term can be multiplicative or additive and may be normally distributed or of any other distribution type, depending on the model.

There are four justifications for using stochastic error terms. First, the omission of some variables in the equation. Second, is the misspecification of the equation for the chosen functional form, i.e., may be linear or non-linear in both the dependent and independent variables. Thirdly, the included variables may be measured inaccurately. Fourthly, there may be basic randomness in the behavior of both independent and dependent variables of the equation.

In this thesis, the error term is additive normally distributed for the share equations and cost equation. The share equations are

$$S_i = \frac{\sum_j \beta_{ij}(w_i w_j)^{\lambda/2}}{\sum_i \sum_j \beta_{ij}(w_i w_j)^{\lambda/2}} + \phi_i \ln y + \tau_i t + \epsilon_i \quad \text{Equation 3.8}$$

and the cost function is

$$c(w, y, t) = [1 + \lambda G(w)]^{1/\lambda} y^{\alpha(y, w)} e^{\tau(t, w)} \quad \text{Equation 3.9}$$

If we adjust the cost for technical change and scale we have

$$\frac{\bar{c}^\lambda - 1}{\lambda} = G(w) + u, \text{ where } \bar{c} = c/e^T y^a \quad \text{Equation 3.10}$$

The transformation of Equation 3.9 is intended to make the error term homoscedastic.

As we know that shares add up to unity, and to make the errors in Equation 3.8 independent, we will have to drop one of the share equations from the system. As in the case of maximum likelihood estimation involving (a wide range of a priori information, pertaining not only to each equation individually, but to several equations simultaneously, such as constraints involving coefficients of different structural equations and certain restrictions on error structure), it is of no consequence to drop one of the share equations because the ML method is invariant to normalization.

Further we assume that ϵ_i and u have joint normal distribution with mean vector zero and covariance matrix Ω . The Ω is non-singular and is assumed to be constant, because any scale effects of the error variance-covariance have been taken care of by the share equations.

The observed endogenous variables are total cost c_i and share S_i , and, as c_i is transformed, the likelihood function will include the absolute value of the determinant of the transformation. This is known as the Jacobian of the transformation. The value of the Jacobian is the matrix containing in its ij th position the derivative of the i th observation of u with respect to the j th observation of c_i , and similarly for ϵ_i .

$$J_t = \begin{vmatrix} \frac{\partial u}{\partial c} & \frac{\partial u}{\partial S_i} \\ \frac{\partial \epsilon_i}{\partial c} & \frac{\partial \epsilon_i}{\partial S_i} \end{vmatrix} \quad \text{Equation 3.11}$$

From Equation 3.9

$$\frac{\partial u}{\partial c} = \tilde{c}_t^{\lambda-1} \cdot \frac{\partial \tilde{c}}{\partial c} = \tilde{c}_t^{\lambda-1} \cdot \frac{1}{e^{\tau y^\alpha}} = \frac{\tilde{c}_t^\lambda}{\tilde{c}_t e^{\tau y^\alpha}} = \frac{\tilde{c}_t^\lambda}{c_t} \text{ and } \frac{\partial u}{\partial S_i} = 0, \quad \frac{\partial \epsilon_i}{\partial c} = 0 \quad \frac{\partial \epsilon_i}{\partial S_i} = 1,$$

and, putting these in above equation, we get

$$J_t = \tilde{c}_t^\lambda / c_t \quad \text{Equation 3.12}$$

The log of the concentrated form of the likelihood function (Klein, 1952) for the observed total cost c_t and shares S_i for T sample observations is given as

$$L(b) = B - \frac{T}{2} \ln |\Omega| + \sum_{t=1}^T \ln(\text{abs } J_t) \quad \text{Equation 3.13}$$

or

$$L(b) = B - \frac{T}{2} \ln |\hat{\Omega}| + \lambda \sum_{t=1}^T \ln \tilde{c}_t - \sum_{t=1}^T \ln c_t ;$$

here,

b = a vector of all the parameters

B = constant term = $-T/2 \ln 2 \pi$

$\hat{\Omega}$ = maximum likelihood estimate of Ω^{MLE}

$\text{abs } J_t$ = absolute value of the Jacobian

$$\hat{\Omega} = \frac{1}{T} \begin{vmatrix} \hat{u}'\hat{u} & \hat{u}'\hat{\epsilon}_1 & \dots & \dots & \hat{u}'\hat{\epsilon}_{n-1} \\ \hat{u}'\hat{\epsilon}_1 & \hat{\epsilon}'_1\hat{\epsilon}_1 & \dots & \dots & \hat{\epsilon}'_1\hat{\epsilon}_{n-1} \\ \vdots & & & & \vdots \\ \vdots & & & & \vdots \\ \hat{u}'\hat{\epsilon}_{n-1} & \dots & \dots & \dots & \hat{\epsilon}'_{n-1}\hat{\epsilon}_{n-1} \end{vmatrix}, \quad \text{Equation 3.14}$$

where

$$u = \frac{\bar{c}^{\lambda-1}}{\lambda} - G(w) = \bar{c}^{\lambda}/\lambda - \frac{2}{\lambda^2} \sum_i \sum_j \beta_{ij}(w_i w_j)^{\lambda/2} \quad \text{Equation 3.15}$$

and

$$\epsilon_i = S_i - \frac{\sum_j \beta_{ij}(w_i w_j)^{\lambda/2}}{\sum_i \sum_j \beta_{ij}(w_i w_j)^{\lambda/2}} - \phi_i \ln y - \tau_i t, \quad i=1 \dots n-1 \quad \text{Equation 3.16}$$

For different values of λ we can get a different functional form; for example when λ tends to zero, in Equation 3.13, we have the likelihood functional form for the translog function.

$$L_o(b) = B - \frac{T}{2} \ln |\hat{\Omega}_o| - \sum_{i=1}^T \ln c_i, \quad \text{Equation 3.17}$$

where

$$\hat{\Omega}_o = \frac{1}{T} \begin{vmatrix} \hat{u}'_o \hat{u}_o & \hat{u}'_o \hat{\epsilon}_{1,0} & \dots & \dots & \hat{u}'_o \hat{\epsilon}_{n-1,0} \\ \hat{u}'_o \hat{\epsilon}_{1,0} & \hat{\epsilon}'_{1,0} \hat{\epsilon}_{1,0} & \dots & \dots & \hat{\epsilon}'_{1,0} \hat{\epsilon}_{n-1,0} \\ \vdots & & & & \vdots \\ \vdots & & & & \vdots \\ \hat{u}'_o \hat{\epsilon}_{n-1,0} & \dots & \dots & \dots & \hat{\epsilon}'_{n-1,0} \hat{\epsilon}_{n-1,0} \end{vmatrix} \quad \text{Equation 3.18}$$

and

$$\begin{aligned}
u_o &= \lim_{\lambda \rightarrow 0} u = \lim_{\lambda \rightarrow 0} \left[\frac{\tilde{c}^\lambda - 1}{\lambda} - G(w) \right] \\
&= \ln \tilde{c} - \lim_{\lambda \rightarrow 0} G(w) \\
&= \ln \tilde{c} - \gamma_o - \sum_i \gamma_i \ln w_i - \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln w_i \ln w_j
\end{aligned}$$

Equation 3.19

$$\begin{aligned}
\epsilon_{i,0} &= \lim_{\lambda \rightarrow 0} \epsilon_i = \lim_{\lambda \rightarrow 0} \left[S_i - \frac{(\gamma_i + \sum_j \beta_{ij} w_j(\lambda)) w_i^{\lambda/2}}{1 + \lambda G(w)} - \phi_i \ln y - \tau_i t \right] \\
&= S_i - \gamma_i - \sum_j \beta_{ij} \ln w_j - \phi_i \ln y - \tau_i t.
\end{aligned}$$

Equation 3.20

From the basic theory of transformations, we know that the likelihood function 3.13 is continuous even at $\lambda=0$; hence the parameter vector (b) can be estimated by maximizing the sample values of this equation.

The necessary and sufficient condition for the function 3.13 to reach a maximum value is

$$(a) \frac{\partial L(b)}{\partial b} = 0 \text{ and } (b) \frac{\partial^2 L(b)}{\partial b^2} < 0$$

Equation (a) is the gradient of the function. Equation (b) is Hessian matrix of the function.

If the value obtained above is a global maximum, then the parameter estimates are consistent, asymptotically efficient and normal.

The asymptotic variance of the maximum likelihood function is usually equal to the Cramer-Rao lower bound, the lowest asymptotic variance that a consistent estimator can have. That is why the maximum likelihood estimate (MLE) is asymptotically efficient.

Consequently, the variance (not just the asymptotic variance) of the MLE estimated by an estimate of the Cramer-Rao lower bound.

That is

$$V(b) = - \left[\frac{\partial^2 L(b)}{\partial b^2} \right]^{-1} \quad \text{Equation 3.21}$$

where the R.H.S. of Equation 3.21 represents the negative inverse of the Hessian matrix for the maximum likelihood estimates.

Testing Design, Criteria and Measures of Fit

The actual properties of the true function are not preserved in the case of approximation by flexible functional forms (L.J. Lau 1974). Approximating a function will induce auto-correlation and hetroscaasticity of an unknown magnitude in the residuals. It also obstructs the desired fit of data points, and bias and dispersion of parameter estimates. Therefore, to avoid this, the unapproximated generalized function is treated in this study.

For testing hypotheses, the F test is applicable whenever we are testing linear restrictions in the context of the classical non-linear regression model. On the other hand if the restrictions are non-linear, the model is non-linear in the parameters, or the errors are not distributed normally, the F test is inappropriate and usually we look into other methods.

Three asymptotically equivalent tests (methods) are:

1. Likelihood Ratio Test (LR)
2. Wald Test (W)

3. Lagrange Multiplier Test (LM)

The test statistics associated with these tests are distributed asymptotically as Chi square (χ^2) with degrees of freedom equal to the number of restrictions being tested, but have unknown small sample distribution. The rationales of these tests are different.

Suppose that $g(\beta)=0$ (in Figure 3.1) at the value of β_R^{MLE} , where the function $g(\beta)$ cuts the horizontal axis. According to the three tests we have:

1. LR Test According to this, if the restriction $g(\beta)=0$ is true, then $\ln L_R$, the maximized value of $\ln L$ imposing the restriction should not be significantly less than $\ln L_{max}$, the unrestricted maximum value of $\ln L$. The LR method tests whether $(\ln L_R - \ln L_{max})$ is significantly different from zero.

2. W Test This test exploits the fact that, if the value of $g(\beta)=0$, then $g(\beta^{MLE})$ (β^{MLE} the unrestricted estimate of β) should not be significantly different from zero. The W method tests whether the difference between $(\beta)^{MLE}$ and β_R^{MLE} is significant or not.

3. LM Test This test considers whether the slope of $\ln L_R$ w.r.t β is different from zero (significantly) at β_R^{MLE} , because the $\ln L$ is maximized at point A, where $\partial \ln L / \partial \beta = 0$.

The choice among these three tests depends upon their small sample distribution properties, computational cost involved and difficulties of estimating the desired statistic. In our case we take the 'LR' test ratio as the criterion of testing the hypothesis, because our cost and share equations are already in the log form. The other reason is, as shown by Berndt and Savin (1977), that for linear models in small sample, the values of these

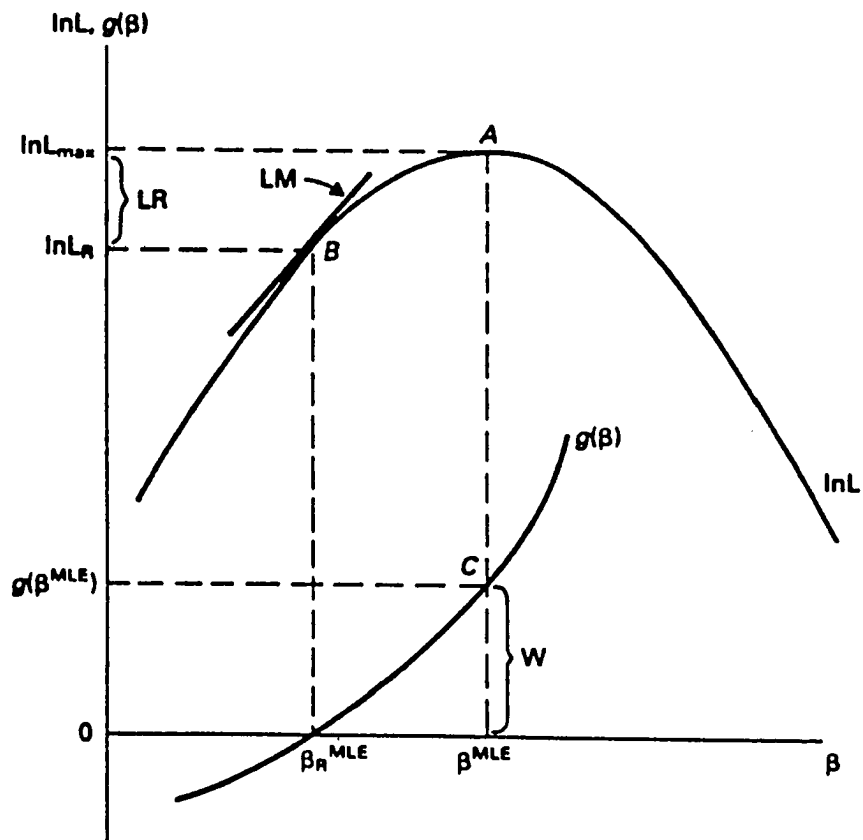


Figure 3.1 Explaining the LR, W and LM statistics (Kennedy, 1985, p. 78)

test statistics are such that $W \geq LR \geq LM$ for the same data and same restrictions. Also it is unbiased and consistent.

The LR test statistic is computed as $-2\ln\lambda$, where λ is likelihood ratio, the ratio of the restricted maximum of the likelihood (i.e., under the null hypothesis H_0) to the unrestricted maximum of the likelihood as

$$\lambda = \frac{\max L(b) \text{ over values of } b \text{ specified by } H_0}{\max L(b) \text{ over all values of } (b)} \quad \text{Equation 3.22}$$

In our case it is $2(L_u - L_r)$, where

L_u = log of likelihood of unrestricted model

L_r = log of likelihood of restricted model.

This statistic is distributed asymptotically as χ^2 with degrees of freedom equal to the number of restrictions in L_r as compared to the number in L_u .

For checking the parameter estimates, the asymptotic 't' ratio test is used. This is the ratio of the parameter estimate to the asymptotic standard error. This 't' ratio test is compared with critical 't' values to test the null hypothesis that the relevant parameter is zero. The square of the asymptotic 't' ratio, which is called Wald statistic, has already been discussed above, and has the χ^2 distribution with 1 degree of freedom.

The value of R^2 is based on the formula by Baxter and Cragg (1970), and is computed to measure the goodness of fit test as follows.

$$R^2 = 1 - e^{2(L_r - L_u)/T}, \quad \text{Equation 3.23}$$

where

L_r = log of likelihood (maximum sample value) when all slope coefficient are zero except γ_0 and λ

CHAPTER IV. MODEL APPLICATION TO PAKISTANI TEXTILES DATA, 1965-1989

In this chapter we use the generalized functional form discussed in the previous chapters to estimate a four-factor model of the Pakistani textile industry. The four factors are capital (K), labor (L), energy (E) and intermediate material (M) for the years 1965-1989.

A Model

We assume that the Pakistani textile industry can be characterized by a production (or cost) function which is continuous, twice differentiable and concave (convex) in the input levels. The inputs are chosen to be at levels, which minimize the total cost of producing the desired output with the given input prices, and behavioral knowledge of the process of production. The time trend 't' in the function is represented as the improvement/efficiency in the process of production, or technical change or progress, which eventually tends to decrease the cost of production.

We can have the minimum total cost function from Chapter III as:

$$C = c(w_i, y, t), \text{ where } w_i = w_K, w_L, w_E, w_M \quad \text{Equation 4.1}$$

and

w_K = price of capital services (rent/interest)
 w_L = price of labor service (wage rate)
 w_E = price of energy (unit price)
 w_M = price of other intermediate materials.

The function in Equation 4.1 is linearly homogeneous and concave in prices, due to the duality theorem discussed in Chapter II.

The Equation 4.1 can be written in the generalized specification as

$$c = [1 + \lambda G(w)]^{1/\lambda} \cdot y^{\alpha(y,w)} e^{T(t,w)} \quad \text{Equation 4.2}$$

where

$$G(w) = \gamma_o + \sum_i \gamma_i w_i(\lambda) + \frac{1}{2} \sum_i \sum_j \gamma_{ij} w_i(\lambda) w_j(\lambda) \quad i, j = K, L, E, M \quad \text{Equation 4.3}$$

$$w_i(\lambda) = (w_i^{\lambda/2} - 1)/\lambda/2 \quad i = K, L, E, M \quad \text{Equation 4.4}$$

$$\alpha(y,w) = \alpha + \left(\frac{\theta}{2}\right) \ln y + \sum_i \phi_i \ln w_i, \quad i = K, L, E, M \quad \text{Equation 4.5}$$

$$T(t,w) = t(\tau + \sum_i \tau_i \ln w_i), \quad i = K, L, E, M \quad \text{Equation 4.6}$$

By imposing symmetry and linear homogeneity restrictions we can write Equation 4.2 as

$$c = \left[\frac{2}{\lambda} \sum_i \sum_j \beta_{ij} (w_i w_j)^{\lambda/2} \right]^{1/\lambda} y^{\alpha(y,w)} e^{T(t,w)}, \quad i = K, L, E, M \quad \text{Equation 4.7}$$

The translog form of Equation 4.7 is obtained by letting $\lambda \rightarrow 0$, and, rearranging the equation, as in Chapter II, we get

$$\ln c = \gamma_o + \sum_i \gamma_i \ln w_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln w_i \ln w_j + \alpha(y,w) \ln y + T(t,w), \quad \text{Equation 4.8}$$

$$i = K, L, E, M$$

The share equations are obtained by differentiation of Equation 4.8 as $\partial \ln c / \partial \ln w_i$

$$S_i = \frac{\sum_j \beta_{ij} (w_i w_j)^{\lambda/2}}{\sum_j \beta_{ij} (w_i w_j)^{\lambda/2}} + \phi_i \ln y + \tau_i t + \epsilon_i, \quad i = K, L, E, M \quad \text{Equation 4.9}$$

where ϵ_i = error of optimizing behavior (stochastic).

$$S_i = \gamma_i + \sum_j \beta_{ij} \ln w_j + \phi_i \ln y + \tau_i t + \epsilon_i, \quad i = K, L, E, M \quad \text{Equation 4.10}$$

As we know that $\sum S_i = 1$, and Equations 4.9 and 4.10 might not satisfy this. To avoid this we can arbitrarily drop any one of the equations to normalize the system. The maximum likelihood estimate is invariant to this effect. We drop the S_M , and the lost degrees of freedom of the system can be recovered by including the cost equations 4.11 and 4.12 with additive stochastic error, which help permit to estimate parameters not included in the share equations:

$$\frac{\bar{c}^\lambda - 1}{\lambda} = \frac{2}{\lambda} \sum_i \sum_j \beta_{ij} (w_i w_j)^{\lambda/2} + u \quad \text{Equation 4.11}$$

or

$$\ln \bar{c} = \gamma_o + \sum_i \gamma_i \ln w_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln w_i \ln w_j + u \quad \text{Equation 4.12}$$

The error vector $e = (u, \epsilon_K, \epsilon_L, \epsilon_E)$ is assumed to be multivariate normally distributed with mean 0, and non-singular variance-covariance matrix of Ω , as estimated in Equation 4.14. The log of the likelihood function is given as

$$L(b) = B - \frac{T}{2} \ln |\hat{\Omega}| + \sum_{i=1}^T \ln \text{abs} |J_i| \quad \text{Equation 4.13}$$

where all the terms have the meanings defined in Chapter III.

As indicated above, for observed residuals (u , ϵ_K , ϵ_L , ϵ_E), the value of Ω is estimated by

$$\hat{\Omega} = \frac{1}{T} \begin{bmatrix} \hat{u}'\hat{u} & \hat{u}'\hat{\epsilon}_K & \hat{u}'\hat{\epsilon}_L & \hat{u}'\hat{\epsilon}_E \\ \hat{u}'\hat{\epsilon}_K & \hat{\epsilon}_K'\hat{\epsilon}_K & \hat{\epsilon}_K'\hat{\epsilon}_L & \hat{\epsilon}_K'\hat{\epsilon}_E \\ \hat{u}'\hat{\epsilon}_L & \hat{\epsilon}_K'\hat{\epsilon}_L & \hat{\epsilon}_L'\hat{\epsilon}_L & \hat{\epsilon}_L'\hat{\epsilon}_E \\ \hat{u}'\hat{\epsilon}_E & \hat{\epsilon}_K'\hat{\epsilon}_E & \hat{\epsilon}_L'\hat{\epsilon}_E & \hat{\epsilon}_E'\hat{\epsilon}_E \end{bmatrix}, \quad \text{Equation 4.14}$$

and

$$J_t = \tilde{c}_t^\lambda / c_t \quad \text{Equation 4.15}$$

The values of the parameters which maximize Equation 4.13 over the sample values are chosen. As we know that the maximum likelihood estimators are consistent, asymptotically normally distributed and efficient, the variance-covariance matrix of the parameter estimates is estimated, as indicated by Equation 3.21, by

$$\hat{V}(b) = - \left[\frac{\partial^2 L(\hat{b})}{\partial b^2} \right]^{-1} \quad \text{Equation 4.16}$$

The right-hand side of Equation 4.16 is also known as the Cramer-Rao lower bound matrix, i.e., a minimum variance bound, the inverse of which is called information matrix. The analysis is done on the Zenith data processing system using Shazam software available in the Economics computing room.

The Data

The data used in this study are the quantities and prices of four input services and the output quantities of the large scale Pakistani Textile Industry from 1965 to 1989. The data is collected from the Census of Manufacturing Industries (CMI), the Pakistan

Statistical Division, the Pakistani Statistical Yearbook, the United Nations Statistical Yearbook, and many others mentioned in the bibliography. The total number of the establishments considered for the analysis are 470 and includes spinning, weaving and finishing of cotton, wool, silk, synthetic and narrow fabric textiles. Hand looms are not included in the analysis.

Definition of the Factors Used

Capital: Capital consists of all the fixed assets of the industry including land and building, plant and machinery and all other assets which are expected to have a production life of more than one year, and are used by the establishment for the manufacturing activity. The used value is determined at the end of the year after taking into account the value of addition and alteration, by deducting the value of sales and loss due to theft or fire and depreciation during the year. The land includes all land owned by the factory at its location in addition to the land on which factory buildings are situated. By buildings is meant all the structures at the factory location used directly or indirectly for the manufacturing process, goods and buildings used for welfare purposes of the workers. Residences and canteens are included. Plant and machinery include machines, tools and other mechanical equipment used in the manufacturing process. Other assets are vehicles, furniture, fixtures and durable spare parts.

Labor: Labor includes production workers who are engaged in production work directly in manufacturing, assembling, packing, repairing, etc. Working supervisors and persons engaged for repair and maintenance are also included. Non-production

employees such as administrative and professional, white collar office employees and contract labor who are engaged through labor contract are not included.

Energy: Fuel like coal, coke, charcoal, firewood, fuel oil, gas, electricity and other such items that are consumed in generating power are called energy. Fuel consumed in generation of electricity is included.

Intermediate Materials: Raw material and semi-finished material, assembling parts, etc., which are physically incorporated in the products and by-products made, chemicals, lubricants and packing materials that are consumed in production, and spare parts which are charged to current operating expenses, are included in intermediate materials. Raw materials given to other establishments for manufacturing goods (semi-finished and finished) on behalf of the establishment are also included. The raw material supplied by others for manufacturing goods on their behalf, though, is excluded.

Output: Output is the value of the production and it includes the value of the items produced or manufactured in the manufacturing process. In addition to the value of the individual items produced, data regarding value of by-products, wastes, electricity generated and sold (if any) and of fixed assets produced by the establishment for its own use (if any) are also outputs. Valuation of individual items of products and by-products is made at the ex-factory price which includes indirect taxes (excise duty, sales tax, etc.) and excludes transport cost outside the factory gate.

From the data it can be seen that demand for energy and capital increased in line at a faster rate than labor, but the price of labor increased at a higher rate than capital and energy. The average productivity of labor increased from 4.35 in 1965 to 6.65 in 1989,

while the average productivity of capital decreased from 22.26 to 18.24 in the same period. The average productivity of energy is 25.29 in 1965 and 25.58 in 1989, which means that it remained virtually unchanged during this period.

The inconsistency in input demands, seen in the data can be attributed to the following factors:

1. own price elasticity effect (σ_{ii}),
2. substitution effect, one factor for the other (σ_{ij})
3. non-homothetic return to scales, i.e., $\phi_i \neq 0$.
4. non-neutral technical change, i.e., $\tau_i \neq 0$.

The analysis in this study is focused on these factors. In the case of the generalized function, the above factors, and therefore, elasticities of substitution, returns to scales and technical change bias, can be determined simultaneously.

The results of this study are presented below in detail in the following orders:

- (a) Selection of models of technology,
- (b) Selection among alternative functional forms,
- (c) Technical change and scales effects,
- (d) Estimated cost function,
- (e) Price and substitution elasticities,
- (f) Total factor productivity and return to scale, and
- (g) Effects of changes in input price, scale and time.

Selected Models of Technology

In the generalized function, various models of technology can be estimated. This estimation depends upon the nature of returns to scale and technological change. Three types of technical change will be assumed, i.e., no change, neutral and non-neutral technical change, and four types of returns to scale, i.e., constant, homogeneous and non-homogeneous but homothetic and non-homothetic returns to scale. Therefore, there are 12 different combinations possible. Table 4.1 shows the classification of the 12 different model combinations.

Figure 4.1 represents the sample values of log of likelihood for the combinations given in Table 4.1. The general model with no restrictions is NH-NNE, in which there are 20 free parameters. The most restricted one is CRTS-N with eleven free parameters.

Table 4.2 represents the results of Chi square test statistics. Part (a) is for testing various kinds of scales returns, while part (b) is for testing technical change effects under various model restrictions. From table (a) it is clear that whatever may be the type of the technical change, the hypothesis of homotheticity is totally rejected because the 1% critical value of the χ^2 test statistics are much smaller than the observed value. Even if homotheticity may be imposed mistakenly, the further restriction of homogeneity does not make any difference to the structure with the same level of confidence. The restriction of constant returns to scale, with given homotheticity, doesn't result in significant loss of fit, except in the case of the no change alternative.

Table 4.1 Classification of models

Type of Returns	Technical Change		
	No Change (N)	Neutral Change (NE)	Non-neutral Change (NNE)
Constant returns to scales (CRTS)	CRST-N	CRST-NE	CRTS-NNE
Homogeneous Homothetic (HH)	HH-N	HH-NE	HH-NNE
Non-homogeneous Homothetic (NHH)	NHH-N	NHH-NE	NHH-NNE
Non-Homothetic (NH)	NH-N	NH-NE	NH-NNE

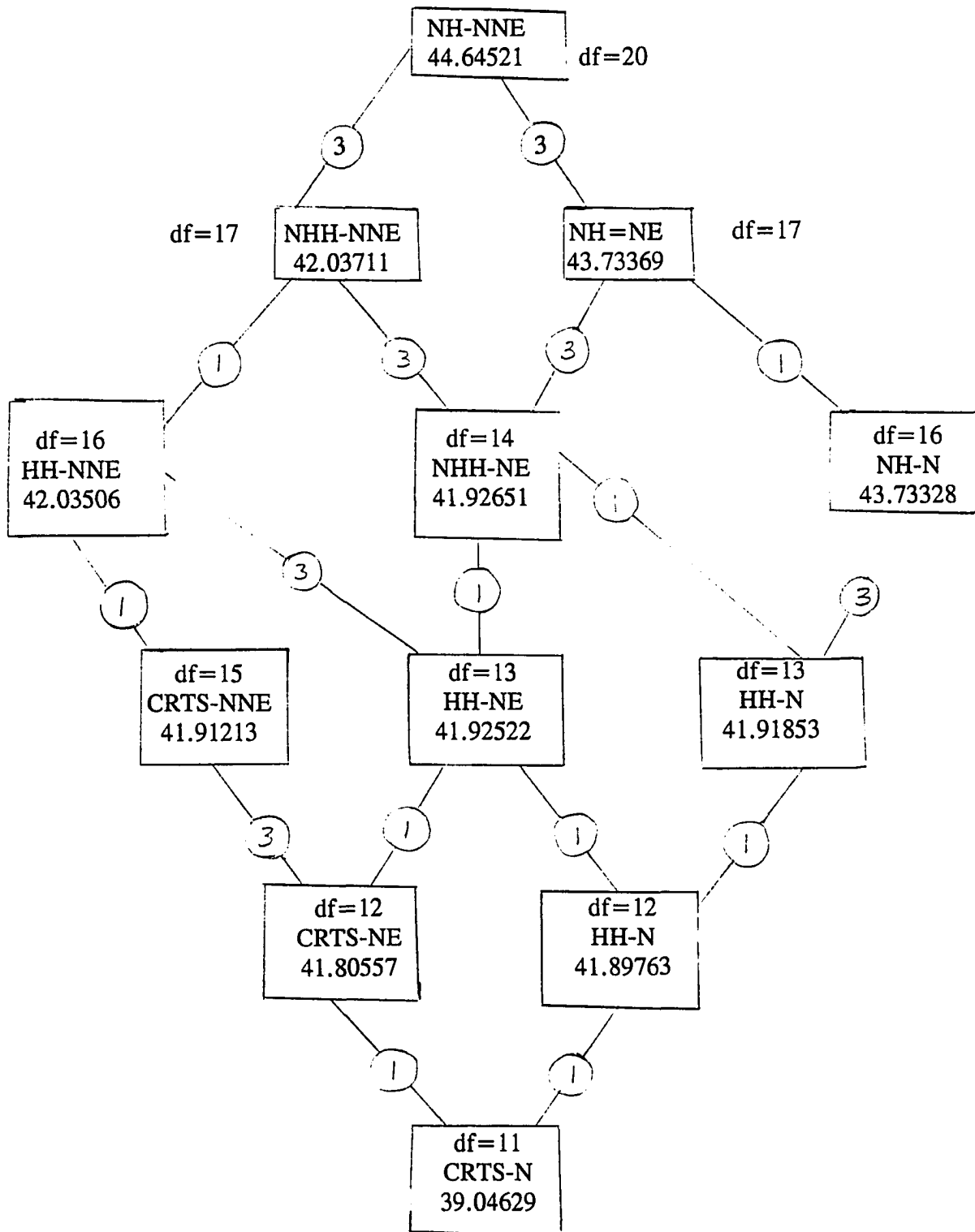


Figure 4.1 Maximum sample log of likelihood values (circle indicates the number of restrictions)

Table 4.2 χ^2 Test statistics for selected models(a) Returns to scale

	Technical Change		
	No	Neutral	Non-neutral
Homotheticity	32.3210* (11.344)	33.1839* (11.344)	49.1732* (11.344)
Homogeneity given homotheticity	0.4988 (6.6349)	0.0389 (6.6349)	0.0499 (6.6349)
CRTS given homotheticity	54.8437* (9.21)	2.8936 (9.21)	2.5889 (9.21)

(b) Technical change

	CRTS	HH	NHH	NH
Neutrality	2.2823 (11.344)	2.3868 (11.344)	2.6726 (11.344)	16.8809* (11.344)
No change given neutrality	53.9858* (6.6349)	0.5698 (6.6349)	0.1885 (6.6349)	.00795 (6.6349)

*shows that values are significant at 1% level. Values in parenthesis are 1% critical values of χ^2 for 3 d.f. (as per Figure 4.1); the statistics are computed as $2(L_u - L_r)$

From part (b) of the table it can be seen that neutrality of technical change is only rejected in the non-homothetic case. However, if neutrality is assumed, then further restriction of no technical cannot be rejected, except in the case of constant returns to scale.

This shows that the Pakistani textile industry significantly exhibits non-homothetic returns to scale and non-neutral technical change. However, with the assumption of homotheticity and neutrality, we can't reject CRTS-NE and HH-N, the simplest models, when tested against the next simplest hypothesis. In these two models productivity growth is attributed to the technical progress in the first and to scale economies in the second model. As both models are nested in the HH-NE model (see Figure 4.1), the comparison shows no significant loss of fit for one degree of χ^2 , and hence fails to decide whether the productivity growth is due to technological change or to scale economies. As the third (HH-NE) model is dominated by NH-NNE model, in the next section we will compare the NH-NNE model with the HH-NE, HH-N and CRTS-NE models, for proper evaluation of the results, because we believe that the returns to scale and technological development are non-homothetic and non-neutral.

Functional Form Alternatives

In Table 4.3 we have presented the sample maximum log likelihood values for different models. The table shows the values of LnL when λ is held free to be estimated as $\hat{\lambda}$. The asymptotic t ratio defined in Chapter III are in parenthesis. In Table 4.4, the results of the hypothesis $H_0: \lambda=i$, when $i=1,2,0,-1,-2$ verses $H_A: \lambda \neq i$ are presented for the models in Table 4.3.

Table 4.3 Sample maximum log of likelihood values for selected models

(a) when λ is free

Models	NH-NNE	HHNE	CRTS-NE	HH-N
$\hat{\lambda}$	0.65	-.89	-.78	-.87
t	(5.681)	(-2.829)	(-2.944)	(-2.842)
LnL	44.64521	41.9255	41.80557	41.89763

(b) when λ is fixed

λ	Models	NH-NNE	HHNE	CRTS-NE	HH-N
2	LnL	43.8363	41.60754	41.2385	41.5489
1		44.5699	41.6261	41.4876	41.62512
0		43.9898	41.7880	41.6854	41.7668
-1		43.6158	41.9238	41.8024	41.8954
-2		43.0086	41.3109	41.2510	41.1899

NOTE: 't' is the asymptotic t ratio = parameter estimate/asymptotic standard error

t^2 is known as the Wald statistic and is distributed as χ^2 with one degree of freedom under the null hypothesis.

In the case of the NH-NNE model, it is clear that $\lambda=1$ cannot be rejected at the one percent level; recall that $\lambda=1$ represents the Generalized Leontief function (see Chapter II). In the case of the other models, only $\lambda=0$ (Translog) and $\lambda = -1$, can't be rejected at the five percent level.

From Table 4.3(a) we can see that Pakistani textiles can be represented by the generalized function when the value of $\lambda = 0.65$. However, it can be represented by the values of $\lambda = -1$, $\lambda=0$ (Translog) and $\lambda=1$ (Generalized Leontief) at the five percent level without any loss of fit, if homotheticity and neutrality are assumed (Table 4.3(b)). It is worth noting that $\lambda=1$ gives the representation of the generalized form, which of course includes non-neutrality and non-homotheticity.

Technological Change and Scale Economies

Table 4.5 gives the maximum likelihood estimates of the parameters. Our accepted NH-NNE model is bifurcated into NH-NNE'A' and NH-NNE'B'. In the model 'A' we restrict the value of ϕ and τ to 0, while in model 'B' we restrict λ to 1 in addition to the restrictions of model A. For the case of the NH-NNE model, the values of ϕ_K , ϕ_L and ϕ_E are significantly negative, which means that scale economies are possible with increased output. From Chapter III we know that when $\phi_i < 0$, the effect is share saving. Hence the scale effect in the Pakistani textile industry is share saving for labor, capital and energy.

Similarly the technical change effects are share-using for capital and energy and share-saving for intermediate product, while approximately share-neutral for labor as is clear by τ_K , τ_E , τ_M and τ_L . However, this is not inconsistent with the finding, in general,

Table 4.4 Chi square test for alternative form in the case of selected models

λ	NH-NNE	HH-NE	CRTS-NE	HH-N
2	18.2345	5.8569*	12.3143	7.3123
1	1.6735*	5.6784*	6.2893*	5.7812*
0	12.7897	3.0410*	2.9478*	2.3173*
-1	19.6781	.0379*	0.735*	.0535*
-2	31.3497	11.3767	12.0874	13.1347

* indicates that at 1% level the null hypothesis cannot be rejected.

Chi square critical value with one degree of freedom at 1% level = 6.6349

Chi square critical value with one degree of freedom at 5% level = 3.841

Table 4.5 Maximum likelihood parameter estimates in selected models (asymptotic t ratios are in parentheses)

	NH-NNE	NH-NNE'A'	NH-NNE'B'	HH-NE	HH-N	CRTS-NE
β_{KK}	-0.55 (-1.883)	-.028 (-1.117)	-.076 (-7.231)	.023 (2.253)	.017 (3.598)	.045 (6.081)
β_{KL}	.427 (1.404)	.232 (3.306)	.312 (4.493)	-.005 (-2.000)	-.003 (-3.775)	0.12 (-1.776)
β_{KE}	.124 (1.342)	.064 (2.878)	.09 (5.677)	-.003 (-2.160)	-.005 (-2.316)	-0.10 (-3.419)
β_{KM}	-.136 (-1.1420)	-.076 (-1.658)	-.050 (-8.28)	-0.23 (-2.134)	.017 (-3.883)	0.047 (-3.463)
β_{LL}	.523 (1.454)	.289 (3.300)	.292 (2.835)	.091 (2.551)	.066 (9.504)	.163 (2.990)
β_{LE}	.302 (1.460)	.176 (3.692)	.223 (5.059)	-.012 (-2.333)	-.008 (-6.264)	-.017 (-2.169)
β_{LM}	-.631 (-1.461)	-.343 (-3.363)	-.319 (-2.708)	-.141 (-2.491)	-.103 (-14.8)	-.249 (-2.469)
β_{EE}	-.112 (-1.302)	-.061 (-2.556)	-.093 (-5.587)	.020 (2.147)	.013 (3.95)	.035 (3.60)
β_{EM}	-0.65 (-1.975)	-.040 (-1.259)	-.017 (-4.49)	-.016 (-1.720)	-.011 (-2.741)	-.031 (-2.21)
β_{MM}	0.912 (1.487)	.501 (4.020)	.502 (3.596)	.012 (.404)	.0062 (.349)	.025 (.63)
α	.601 (2.236)	.824 (78.63)	.823 (79.33)	.867 (11.05)	.899 (102.33)	1.00
τ	.001 (.832)	0.0	0.0	-.002 (-.742)	0.0	-.0065 (-16.74)
θ	0.34 (.724)	0.0	0.0	0.0	0.0	0.0

of labor saving and capital and energy using in manufacturing by the Pakistani Industry (Khan 1987).

The value of θ (Table 4.5) is not significant and is positive, which indicates that a minimum point of the average cost curve is reached with increase in output. Also, the value of τ (.001) is not significantly different than zero. In the absence of bias in technical change τ , τ represents the neutral rate of total cost diminution. Hence our NH-NNE'A' model is estimated with $\tau=\theta=0$ and is represented in Table 4.5. The model NH-NNE'B' is also given in the table and estimated with $\theta=\tau=0$ and $\lambda=1$. In both these models we can see from the table that these restrictions do not result in any significant loss of fit. The test statistic χ^2 is .9526 and 1.8354 when both models are compared with the NH-NNE model. The critical value at the 1% level with 2 degrees of freedom is 9.21. Hence our conclusion concerning scalar effects and technological change remain the same. However, it looks useful that the estimate of the parameter estimates are more precise than in the general model, because the standard errors are small.

In the case of the CRTS-NE model, all the growth of productivity is attributed to the neutral technical change effect. The estimated rate of cost diminution of .65% is significantly different from zero because the 't' ratio is 16.74. For the HH-N model, all the increase in productivity is attributed to economies of scale. The scale parameter α is significantly different from 1 because the 't' ratio for $H_0: \alpha=1$ is -22.90. The estimate of $\alpha=0.899$ implies that the estimated degree of homogeneity of the underlying

production function ($1/\alpha$) in input quantities is $1/0.899 = 1.112$, i.e., a contribution of about 11% to the growth attributable to scale economies.

The R^2 values, which are all above .97, reflect that the model explains the data quite well.

Properties of Estimated Cost Function

Based on our maintained hypothesis of linear homogeneity in prices and symmetry of substitution effect, the estimated cost function exhibits positive and monotonic behavior in prices. This all is indicated by the positive values of output elasticities of cost at every point of observation.

For strict concavity of the cost function, the matrix of Allen partial elasticities should be negative definite. This condition is satisfied for the model. This has been checked by showing that the negative of the matrix is positive definite, as verified by criterion (3b) on p. 318 of Graybill (1969). Also, because of the linear homogeneity, the matrix is symmetric.

The concavity violation in the general representation of models NH-NNE'A' and the NH-NNE'B' are not significant in the first half of the sample. The estimates of β_{MM} (0.01 to .03) in the restrictive models (CRTS-NE, HH-N, HHNE) in which concavity is satisfied, is much smaller than those in the NH-NNE'A' and NH-NNE'B' models (.5). For sensitivity analysis, a value of $\beta_{MM}=0.4$ is used, which does not result in any loss of fit at the 1% level ($\chi^2=.7$ only). Also, in the last half of the sample the concavity requirements are satisfied.

Substitution and Price Elasticities

Table 4.5 shows the Allen partial elasticities of substitution based on likelihood parameter estimates. In Table 4.6, the Allen partial elasticities of substitution are given, for the mid point of the data (1977), in our sample. This is done to see what is happening inside the data.

It is clear from the table that elasticity of substitution of capital with respect to energy, i.e., σ_{KE} , varies from -1.97 to -2.578. As the elasticity is negative, it shows that energy resource is acting as a complementary resource with capital. This finding is not inconsistent with the results presented by Khan, A. (1987). On the other hand there are reasonable possibilities of substitution between capital and labor, labor and energy, and material and energy, in all of the selected models. Their ranges are .98 to 1.72 for σ_{KL} , .53 to 1.89 for σ_{LE} and .34 to .89 for σ_{EM} . In the first one-third of the sample, the complementarity nature holds between σ_{KL} , σ_{LE} and σ_{EM} , but later on, it changes to substitutability of the respective input factors.

Table 4.7 represents the cross price elasticities and own price elasticities for four factor inputs. The own price elasticities for L, K, M, and E under the model NH-NNE'A' are -.153, -.329, -.0019 and -.61. It looks like energy demand is most sensitive to its own price.

From the cross price elasticities between labor and energy and labor and capital, it is clear that an increase in the price of energy or capital will increase the demand for labor. The respective elasticities are $\epsilon_{LE} = .084$ and $\epsilon_{LK} = 0.102$. On the other hand an increase in the price of labor will increase the demand for energy and capital at

Table 4.6 Allen partial elasticities of substitution in Pakistani textiles for year 1977
in the case of the selected models

	NH-NNE'A'	NH-NNE'B'	HH-NE	HH-N	CRTS-NE
*					
σ_{KK}	-6.351	-6.634	-8.679	-9.173	-8.345
σ_{KL}	1.692	1.723	1.111	1.123	0.989
σ_{KE}	-2.379	-2.358	-2.410	-2.578	-1.975
σ_{KM}	.0058	.076	.513	.539	.468
σ_{LL}	-.548	-.613	-1.592	1.678	-1.599
σ_{LE}	1.893	1.349	0.601	0.536	0.563
σ_{LM}	-.0519	.023	.603	.589	.597
σ_{EE}	-13.270	-11.37	-11.97	-12.92	-11.324
σ_{EM}	.349	.419	.829	.891	.761
σ_{MM}	-.0025	-.0438	-.375	-.389	-.367

$$*\sigma_{ii} = \frac{\epsilon_{ii}}{S_i}, \sigma_{ij} = \frac{\epsilon_{ij}}{S_j} \quad \epsilon_{ij} = \frac{\partial \ln x_i}{\partial \ln w_j}$$

Table 4.7 Input demand price elasticities in Pakistani textiles in selected models for 1977

	NH-NNE'A' ($\theta=\tau=0$)	NH-NNE'B' ($\tau=\theta=0, \lambda=1$)	HH-NE	HH-N	CRTS-NE
ϵ_{KK}	-.329	-.382	-.435	-.493	-.419
ϵ_{KL}	.479	.453	.258	.269	.272
ϵ_{KE}	-.112	-.110	-.109	-.116	-.097
ϵ_{KM}	.004	.053	.312	.35	.29
ϵ_{LK}	.102	.097	.0535	.0547	.054
ϵ_{LL}	-.153	-.169	-.449	-.457	-.45
ϵ_{LE}	.084	.063	.023	.024	.025
ϵ_{LM}	-.031	.012	.365	.369	.368
ϵ_{EK}	-.135	-.143	-.132	-.152	-.110
ϵ_{EL}	.533	.394	.150	.146	.149
ϵ_{EE}	-.610	-.52	-.54	-.565	-.497
ϵ_{EM}	.217	.263	.522	.567	.459
ϵ_{MK}	.00032	.0042	.021	.027	.024
ϵ_{ML}	-.014	.0058	.159	.164	.165
ϵ_{ME}	.016	0.189	.0359	.042	.319
ϵ_{MM}	-.0019	-.0291	-.227	-.229	-.222

substantial higher rates, because $\epsilon_{EL} = 0.533$ and $\epsilon_{KL} = .479$. Also the complementarity of capital and energy is obvious as shown by their values of $\epsilon_{KE} = -.112$ and $\epsilon_{EK} = -0.135$. Similar results can be obtained in the other selected models.

Table 4.8 gives the various elasticities estimates for 1989, based on our NH-NNE'A' model. In this case energy-capital complementarity is obvious because σ_{KE} is equal -5.465. Also, from cross price elasticity estimates of capital and energy, ϵ_{KE} and ϵ_{EK} indicate that if the price of energy or capital is increased by 1%, the quantity demanded of the other inputs will fall by 0.265%. The results for other elasticities are similar to those obtained above.

Total Factor Productivity and Returns to Scale

Table 4.9 gives the scale effects and cost diminution in selected models. These results are based on the equation discussed in Chapter II. In the model NH-NNE'A', there are significant scale economies and estimated growth of output is about 11%. The growth attributed to scale economies in the case of the HH-NE and HH-N models are 10 and 12% respectively. We reject the hypothesis of constant returns to scale decisively for these models because the asymptotic t ratios are 11.13 and 17.62. This result is in close agreement with the results obtained by Kemmell (1978).

Total factor productivity growth, i.e., technological progress, has contributed very little to the growth of output. The rates of growth for all selected model presented in Table 4.9 are .031%, 0.2% and 0.752%. This estimate is not significantly different from zero for the NH-NNE'A' model, but for the CRTS-NE model, 0.752% growth in TFP is significantly different from zero, as is indicated by the small standard error. The rate of

Table 4.8 Substitution and price elasticities in Pakistani textiles 1989 based in the NH-NNE'A' model

Allen Partial Elasticities of Substitution		Price Elasticities			
σ_{KK}	-3.752	ϵ_{KK}	-0.181	ϵ_{MK}	0.012
σ_{KL}	0.890	ϵ_{KL}	0.259	ϵ_{ML}	0.0265
σ_{KE}	-5.465	ϵ_{KE}	-0.256	ϵ_{ME}	0.0295
σ_{KM}	0.281	ϵ_{KM}	0.169	ϵ_{MM}	-0.069
σ_{LL}	-0.531	ϵ_{LK}	0.045		
σ_{LE}	0.379	ϵ_{LL}	-0.157		
σ_{LM}	0.0859	ϵ_{LE}	0.0649		
σ_{EE}	-11.897	ϵ_{LM}	0.0492		
σ_{EM}	0.649	ϵ_{EK}	-0.259		
σ_{MM}	-0.119	ϵ_{EL}	0.417		
		ϵ_{EE}	-0.556		
		ϵ_{EM}	0.395		

Table 4.9 Returns to scale, cost diminution and TFP in selected models: 1989 for Pakistani textiles

Parameter	NH-NNE'A'	HH-NE	HH-N	CRTS-NE
Returns to Scale (ϵ^*)	1.1183 (0.02036)*	1.102 (0.10378)	1.1217 (.01239)	1.000
Rate of Total Cost Diminution (d)	0.00028 (0.00054)	0.00189 (0.00350)	0	0.00752 (.00058)
Total Factor Productivity	0.000313	0.00208	0	0.00752
TFP= ϵ^*d	(0.00062)	(.00301)		(.000352)

*Number in the parenthesis is the asymptotic standard error. It is calculated by the first order approximation method discussed in Klein (1953, p. 278).

technical progress presented by Khan (1987) is 3.7% per year for all manufacturing industries of Pakistan (textile include). This shows that textile's share of technical progress will be near one percent per year. Hence there is not much discrepancy in our result, when compared to Khan.

With the passage of time, the contribution of scale economies and technical change to growth become more complex. As our data is based on aggregate time series, the results should be interpreted and handled very carefully. Also, if technical progress is dependent upon 'learning by doing,' then as discussed before, the contribution of scale economies to the output growth will increase with time.

Residual Measure of Total Factor Productivity

The residual measure of TFP is defined as output divided by the index of all inputs. The total factor productivity and rate of technical change are as discussed in Chapter II. Thus for the function $y=f(x_i)$, the total factor productivity is given as

$$TFP = y/x, \quad x = \text{index of inputs } x_i \quad \text{Equation 4.17}$$

Equation 4.17 is only valid when (a) the restriction of CRTS is implied, (b) the inputs have been properly measured (real values), (c) there is no mis-specification of production function, (d) TFP is linearly homogeneous and (e) the technical change is Hicks neutral.

Let us assume that the total factor productivity can be specified properly by $TFP = e^{\alpha t}$, the function that gives a neutral rate of technical change. Equation 4.18

Differentiating Equation 4.17 logarithmically w.r.t. time 't' we get

$$\frac{\partial \ln(TFP)}{\partial t} = \frac{\partial \ln y}{\partial t} - \frac{\partial \ln x}{\partial t} = \tau e^{\tau t} \quad \text{Equation 4.19}$$

Divide Equation 4.19 by Equation 4.18 we get

$$\frac{\dot{TFP}}{TFP} = \dot{y}/y - \dot{x}/x = \tau, \text{ where dot denotes the derivative.} \quad \text{Equation 4.20}$$

Here x = aggregate of the inputs (K, L, M, E), which can be obtained as a Divisia Index. (Solow (1957), Denison (1974)). However, we here take the following alternative approach, with results given in p. 123.

From the cost function we can calculate TFP as follows. Let the cost function be written in terms of aggregate input of prices as

$$c = a(t)f(w_K, w_L, w_M, w_E) \quad \text{Equation 4.21}$$

In a similar manner as in Equation 4.19 we have

$$\frac{\dot{a}}{a} = \frac{\dot{w}}{w} - \frac{\dot{c}}{c}$$

where

$$-\frac{\dot{a}}{a} = TFP \text{ (Cost decrease due to technical change)} \quad \text{Equation 4.22}$$

and,

w = aggregate of input,

If we assume a translog cost function we can calculate 'w' by the Tornquist price index as explained by Chambers R.G. (1990, p. 235).

In the case of a competition assumption, the output price equals the unit cost and consequently the TFP can be measured as

$$-\frac{\dot{a}}{a} = \frac{\dot{w}}{w} - \frac{\dot{q}}{q},$$

Equation 4.23

where

q = output price index

For our model, which is based on aggregate data, we use the aggregator function index introduced by Diewert (1976). The aggregator price index is given by

$$w = \left[\frac{\sum_i S_{i,t-1} \left(\frac{w_{it}}{w_{i,t-1}} \right)^{\lambda/2}}{\sum_i S_{it} \left(\frac{w_{i,t-1}}{w_{it}} \right)^{\lambda/2}} \right]^{1/\lambda}$$

Equation 4.24

where the S_i 's are the shares.

Equation 4.24 can be simplified to the Tornquist Index by the Box-Cox transformation, as

$$w^\lambda = \frac{\sum_i S_{i,t-1} \left(\frac{w_{it}}{w_{i,t-1}} \right)^{\lambda/2}}{\sum_i S_{it} \left(\frac{w_{i,t-1}}{w_{it}} \right)^{\lambda/2}} \quad \text{or}$$

$$\frac{w^\lambda - 1}{\lambda} = \frac{1}{2} \frac{\left[\sum_i S_{i,t-1} \frac{\left(\frac{w_{it}}{w_{i,t-1}} \right)^{\lambda/2} - 1}{\frac{\lambda}{2}} - \sum_i S_{it} \frac{\left(\frac{w_{i,t-1}}{w_{it}} \right)^{\lambda/2} - 1}{\frac{\lambda}{2}} \right]}{\sum_i S_{i,t-1} \left(\frac{w_{i,t-1}}{w_{it}} \right)^{\lambda/2}}$$

Equation 4.25

using L'Hospitals' rule and taking the limit in Equation 4.25 as $\lambda \Rightarrow 0$,

$$\begin{aligned}
lnw &= \frac{1}{2} \left\{ \sum_i S_{i,t-1} \ln \frac{w_{it}}{w_{i,t-1}} - \sum_i S_{it} \ln \frac{w_{i,t-1}}{w_{it}} \right\} \\
&= \sum_i \frac{1}{2} (S_{it} + S_{i,t-1}) \ln \frac{w_{it}}{w_{i,t-1}}.
\end{aligned}$$

Equation 4.26

To evaluate the Equation 4.24 we need an estimate $\hat{\lambda}$. In Table 4.3 this estimate $\hat{\lambda} = -0.78$ in our CRTS-NE model. For this value of $\hat{\lambda}$, the residual measures of TFP obtained are presented in Table 4.10 along with the parametric measure for the sake of comparison.

From Table 4.10, one can see that all the residual measures of total factor productivity are nearly identical, except in the competitive cost case, where the discrepancy can be attributed to the difference between the unit cost and output price.

Similarly the residual measure of TFP are nearly close enough to the parametric measures for our CRTS models, because both types of measure of TFP are based on the assumption of constant returns to scale and neutral and homogeneous technical change. The differences between the estimates can be attributed to the inefficiency of the residual measure to account for the cost minimizing behavior. In short, both the CRTS parametric estimates and the residual measure attribute the contribution of scale economy to total factor productivity.

Comparative Statics

Now we consider some sensitivity analysis regarding the effect of technical change and scale of production on parameters like input price, output quantity and time.

Table 4.10 Mean and standard deviation of total factor productivity of Pakistani textile industry (1965-1989)

I. Residual Measure	Mean	Standard Deviation
(a) Tornquist Index ($\lambda=0$) (Equation 4.26)		
(i) From Production Function ($\dot{T}\dot{F}P/TFP$)	0.006212	0.01134
(ii) From Cost Function ($-\dot{a}/a$)	0.006132	0.01128
(iii) From Competitive Cost (\dot{a}/a)	0.006518	0.01326
(b) Aggregator Function Index (Equation 4.24, $\hat{\lambda} = -0.78$)		
(i) From Production	0.006189	0.01232
(ii) From cost	0.006151	0.01192
(iii) From Competitive Cost	0.006525	0.01319
II. Parametric Measure		
(i) CRTS-NE	0.00752	0
(ii) CRTS-NNE	0.00782	.00013
(iii) HH-NE	0.00208	0
(iv) NH-NNE'A'	0.000313	0.00016

Table 4.11 presents the change in respective share as a function of the change of each of the factor inputs, output quantity and time.

The last row of Table 4.11 shows that the signs of the partials derivatives are negative for labor and material share, while for capital and energy they are positive. This means that technological progress tends to increase the capital and energy share, while the shares of labor and material are decreased. This reflects that technical change is labor and materials saving and capital and energy using. From the second-to-last row of the table it can be seen that the increase in output y will decrease the share of all inputs but material. This shows that the economies of scale are not distributed evenly among all inputs.

The 4x4 matrix depicted in Table 4.11 is a symmetric matrix. The cross price effect on the shares is also symmetric. The diagonal elements of the matrix are all positive and represent the own price elasticity of the respective inputs. As elasticity of inputs is positive, any increase in input price will increase the value of the respective share. On the other hand, due to cross price elasticity, it will decrease the value of other shares in the case of capital. A similar effect can be seen for material input. This shows that the price elasticity of substitution between capital and other inputs or between material and other inputs is less than unity. For energy and labor, the growth in one of the shares will increase the other share, while lowering the share of the remaining two. This is because the price elasticity of substitution between labor and energy is more than unity.

Table 4.11 Effect of change in output, time and input prices on estimated share in the NH-NNE'A' model of Pakistani textiles 1965-1989

	$\frac{\partial S_K}{\partial \ln w_i}$	$\frac{\partial S_L}{\partial \ln w_i}$	$\frac{\partial S_E}{\partial \ln w_i}$	$\frac{\partial S_M}{\partial \ln w_i}$
i=K	0.036	-0.0021	-0.015	-0.023
i=L	-.0021	0.165	0.0049	-0.163
i=E	-0.015	0.0049	0.0189	-0.011
i=M	-0.023	-0.163	-0.011	0.1745
w _i =y	-.041	-0.043	-0.0283	0.114
w _i =t	0.00125	-0.00011	0.00071	-0.0017

Table 4.12 presents the effects of input price changes on cost diminution, returns to scale and total factor productivity. We can see that an increase in the price of respective input will increase the non-homothetic returns to scale of these inputs ($\therefore \partial \epsilon^* / \partial \ln w_i > 0$). An increase in the price of capital or energy will decrease the cost diminution rate and rate of total factor productivity. This is not true for labor and material.

The changes in average factors productivity due to variation in input prices, output and time is shown in Table 4.13. It shows that an increase in capital or energy price will raise the average productivities of both inputs while decreasing the other two. This is not true for the labor or material input average productivities. The similarity between the capital and energy inputs shows the complementarity of the two.

The effect of scale of production shows a positive response in all inputs except material. These responses are considerable for capital and energy, due to their small shares and larger scale economies. The technical change effect shows that the average productivities of capital and energy are decreasing, while for labor and material they are increasing, indicating that the former is energy and capital using and latter is labor and material saving responses to technical progress.

Table 4.12 Effects of input price changes on rate of total cost diminution, returns to scale and total factor productivity, Pakistani textiles NH-NNE'A' 1989

	$\frac{\partial d}{\partial \ln w_i}$	$\frac{\partial \epsilon^*}{\partial \ln w_i}$	$\frac{\partial (TFP)}{\partial \ln w_i}$
i=K	-0.00125	0.603	-0.0014
i=L	0.00011	0.0687	0.00011
i=E	-0.00071	0.0429	-0.00079
i=M	0.0017	0.1734	0.0022

Table 4.13 Effects of input prices, output quantity and time on average productivity⁺

	$\frac{\partial \ln(y/K)}{\partial \ln w_i}$	$\frac{\partial \ln(y/L)}{\partial \ln w_i}$	$\frac{\partial \ln(y/E)}{\partial \ln w_i}$	$\frac{\partial \ln(y/t)}{\partial \ln w}$
i=K	0.169	-0.041	0.249	-0.012
i=L	-0.257	0.151	-0.425	-0.025
i=E	0.249	-0.059	0.551	-0.030
i=M	-.168	-0.49	-0.389	0.071
w _i =y	1.021	0.34	0.799	-0.0049
w _i =t	-0.0249	0.00057	-0.0139	0.0032

⁺Estimation is based on the NH-NNE'A' model.

CHAPTER V. SUMMARY AND CONCLUDING REMARKS

In this chapter the summary of the main results obtained in this study are presented.

The main purpose of this study has been to develop an appropriate functional relationship for the Pakistani textile industry to analyze its performance and productivity. Our main hypothesis is that the Pakistani textile industry can be represented by a twice derivable, homogeneous, concave and monotonic production function or by its dual cost function. All this is based on a flexible functional form. Non-homothetic returns to scale and non-neutral technical change in production technology can be incorporated to identify simultaneously the substitution elasticities, scale economies and bias of technical change.

Chapter II is devoted to investigate the model econometrically. The concept of duality among the production function and cost function is utilized to model the system in forms of its cost function. The generalized Box-Cox function with transformation parameter λ is modified and incorporated into the production (cost) function. The different values of λ give rise to different functional forms. Incorporation of non-homothetic scale economies and non-neutral technical change in the model allow us to consider twelve different forms of the model. The share-using or share-saving bias can be interpreted conveniently by homotheticity and neutrality parameters. The various comparative static expression, like estimated shares (S_i), returns to scale, total factor productivity and average productivity, the changes in input price, scale and time are derived and presented.

IV. This program uses the Fletcher Algorithm for obtaining the maximum likelihood estimates of the parameters.

The data was collected through various issues of the Census of Manufacturing Industries, Pakistan Statistical year books and the Economic Survey of Pakistan. The important point in the data sample is that the demand for energy and capital grew at a faster rate than that of labor, while price of labor (wage rate) increased more than that of capital and energy. The goal of this study is to investigate and explain these phenomena, to see whether or not we can attribute them to particular economic entities, i.e., effect of own and cross price elasticity, scale and technical change effect, etc.

It is not possible to do this job of attribution for every possible effect and its cause, without proper model specification. Thus we specify twelve different models based on the nature of technical change and types of returns to scale, as presented in Table 4.1. The concept of homotheticity of production function and neutrality of technical change is clearly rejected, irrespective of the nature of the technical change. Our main accepted hypothesis is that Pakistani textiles can be characterized by non-homothetic production function and non-neutral technical change. It appears from Table 4.2 that if the restriction of homotheticity is mistakenly imposed (Type II error), we cannot reject models involving constant returns to scale, neutrality, homotheticity (including homogeneity), and zero technical change. Such models include the CRTS-NE model, where all the growth in productivity is attributed to technical change, and the HH-N model, where it is attributed to economies of scale only.

To discriminate among alternatives flexible functional forms, the role of λ (the Box-Cox transformation parameter) is significant. For different values of λ we have different flexible functional forms as discussed in Chapter III. Among the different models, the Translog ($\lambda=1$) the GL($\lambda=1$) and $\lambda=-1$ models are the models which do not result in any loss of fit at the 1% level. The values of λ estimated freely as $\hat{\lambda}$ for the alternative models are different from these values, as shown in Table 4.3.

In the case of the NH-NNE (accepted) model it is clear that technical change in Pakistani textiles is capital and energy using, while the estimated neutrality parameters indicate that technical change is labor and material saving. This result is in line with the study presented by Khan (1987). Our model shows the scale economies to be distributed unevenly among the factors of production, with the largest value for labor. We further modified the NH-NNE model to NH-NNE'A' in which we considered the slope of the average cost curve to be zero (\therefore min cost point is reached, and at that point cost rate is also neutral, i.e., $\tau=\theta=0$). The Chi square test statistics show no significant loss of fit under these restrictions. Another version of the NH-NNE model, NH-NNE'B', is tested and compared without any significant difference. However, the simple models NH-NNE'A' and NH-NNE'B' show more precise parameter estimates, as indicated by smaller standard errors. Our subsequent results are based on this model (i.e., NH-NNE'A').

In the case of the cost function, linear homogeneity in prices and reciprocity (symmetry) are the assumptions we make. The monotonicity of the cost function is obvious from the sample. The concavity/convexity condition of the production/cost

function is satisfied in the last half part of the sample in the case of the NH-NNE'A' model, while for the other selected models it is satisfied over the entire sample.

The resulting substitution elasticity estimates show that capital and energy are mutually complementary, while each one of them is substitutable for labor. The price elasticity estimates shows that energy and capital are more own price inelastic than labor. The cross price elasticities estimates in 1977 for NH-NNE'A' show that if price of labor is increased by 1%, the quantity demanded of energy and capital services will be increased by .417% and .259% respectively (Table 4.8).

The effects of scale and technical change in the case of the NH-NNE'A' model reveal that returns to scales are 1.12 and 1.1 for NH-NNE'A' and HH-N and HH-NE respectively. The returns to scale for NH-NNE'A' is significantly different from unity (CTRS), while the others are not significant as compared to the CTRS model.

The growth rate in NH-NNE'A' is only .03%, which is not very impressive as compared to growth in the case of CTRS-NE, which is 0.75% and is consistent with Khan's (1987) disembodied growth of technical change in the whole manufacturing sector of Pakistani Industries.

Comparison of parametric and residual measures of total factor productivity indicates that they are approximately the same when the restriction of constant returns to scale is imposed. Otherwise there is wide difference in both types of measure.

The effect of changes in input price, output quantity and time with respect to scale economies and total factor productivity are considered. The response is positive for scale economies, while it is negative for total factor productivity.

Lastly it is understood that Pakistani large scale textile industry can better be represented by NH-NNE model for economies of scale, but for technical change it is uncertain. As technical change is a complicated phenomenon and needs extensive modeling and research, we suggest that this can be a topic of further research. This research should be attacked both by engineering modeling approaches as well as by economics.

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